QUESTION 43

Choice A is the best answer. No change is needed because the larger "spherically symmetric" droplets indicate that the flights remedied the problem of smaller deformed droplets mentioned earlier in the passage.

Choices B, C, and D are incorrect because none of these choices refers to the size or shape of the biofuel droplets, which is what made the investigation of combustion and fire on Earth problematic.

QUESTION 44

Choice C is the best answer. No comma is needed in the underlined phrase, which clearly and concisely expresses the improved techniques for fighting fires in space or at future outposts on the Moon and Mars that may result from better combustion-rate models.

Choices A and B are incorrect because the commas are incorrectly separating the prepositional phrases from the noun "techniques." Choice D is incorrect because the pair of commas indicate that the information contained between them is nonessential, which isn't accurate.

Section 3: Math Test – No Calculator

QUESTION 1

Choice D is correct. Combining like terms on each side of the given equation yields $6x - 5 = 7 + 2x$. Adding 5 to both sides of $6x - 5 = 7 + 2x$ and subtracting $2x$ from both sides yields $4x = 12$. Dividing both sides of $4x = 12$ by 4 yields $x = 3$.

Choices A, B, and C are incorrect because substituting those values into the equation $3x + x + x + x - 3 - 2 = 7 + x + x$ will result in a false statement. For example, in choice B, substituting 1 for x in the equation would give $3(1) + 1 + 1 + 1 - 3 - 2 = 7 + 1 + 1$, which yields the false statement $1 = 9$; therefore, x cannot equal 1.

QUESTION 2

Choice A is correct. The line passes through the origin. Therefore, this is a relationship of the form $d = km$, where k is a constant representing the slope of the graph. To find the value of k, choose a point (m, d) on the graph of the line other than the origin and substitute the values of m and d into the equation. For example, if the point (2, 4) is chosen, then $4 = k(2)$, and $k = 2$. Therefore, the equation of the line is $d = 2m$.

Choice B is incorrect and may result from calculating the slope of the line as the change in time over the change in distance traveled instead of the change in distance traveled over the change in time. Choices C and D are incorrect because each of these equations represents a line with a d-intercept of 2. However, the graph shows a line with a d-intercept of 0.

QUESTION 3

Choice A is correct. Multiplying both sides of the equation by 6 results in $6E = O + 4M + P$. Then, subtracting $O + 4M$ from both sides of $6E = O + 4M + P$ gives $P = 6E - O - 4M$.

Choice B is incorrect. This choice may result from solving for $-P$ instead of for P. Choice C is incorrect and may result from transposing P with E in the given equation rather than solving for P. Choice D is incorrect and may result from transposing P with E and changing the sign of E rather than solving for P.

QUESTION 4

Choice C is correct. Since $RT = TU$, it follows that ΔRTU is an isosceles triangle with base RU. Therefore, $\angle TRU$ and $\angle TUR$ are the base angles of an isosceles triangle and are congruent. Let the measures of both $\angle TRU$ and $\angle TUR$ be $t°$. According to the triangle sum theorem, the sum of the measures of the three angles of a triangle is 180°. Therefore, $114° + 2t° = 180°$, so $t = 33$.

Note that $\angle TUR$ is the same angle as $\angle SUV$. Thus, the measure of $\angle SUV$ is 33°. According to the triangle exterior angle theorem, an external angle of a triangle is equal to the sum of the opposite interior angles. Therefore, $x°$ is equal to the sum of the measures of $\angle VSU$ and $\angle SUV$; that is, $31° + 33° = 64°$. Thus, the value of x is 64.

Choice B is incorrect. This is the measure of $\angle STR$, but $\angle STR$ is not congruent to $\angle SVR$. Choices A and D are incorrect and may result from a calculation error.

QUESTION 5

Choice B is correct. It is given that the width of the dance floor is w feet. The length is 6 feet longer than the width; therefore, the length of the dance floor is $w + 6$. So the perimeter is $w + w + (w + 6) + (w + 6) = 4w + 12$.

Choice A is incorrect because it is the sum of one length and one width, which is only half the perimeter. Choice C is incorrect and may result from using the formula for the area instead of the formula for the perimeter and making a calculation error. Choice D is incorrect because this is the area, not the perimeter, of the dance floor.

QUESTION 6

Choice B is correct. Subtracting the same number from each side of an inequality gives an equivalent inequality. Hence, subtracting 1 from each side of the inequality $2x > 5$ gives $2x - 1 > 4$. So the given system of inequalities is equivalent to the system of inequalities $y > 2x - 1$ and $2x - 1 > 4$, which can be rewritten as $y > 2x - 1 > 4$. Using the transitive property of inequalities, it follows that $y > 4$.

Choice A is incorrect because there are points with a y-coordinate less than 6 that satisfy the given system of inequalities. For example, (3, 5.5) satisfies both inequalities. Choice C is incorrect. This may result from solving the inequality $2x > 5$ for x, then replacing x with y. Choice D is incorrect because this inequality allows y-values that are not the y-coordinate of any point that satisfies both inequalities. For example, $y = 2$ is contained in the set $y > \frac{3}{2}$; however, if 2 is substituted into the first inequality for y, the result is $x < \frac{3}{2}$. This cannot be true because the second inequality gives $x > \frac{5}{2}$.

QUESTION 7

Choice B is correct. Subtracting 4 from both sides of $\sqrt{2x+6} + 4 = x + 3$ isolates the radical expression on the left side of the equation as follows: $\sqrt{2x+6} = x - 1$. Squaring both sides of $\sqrt{2x+6} = x - 1$ yields $2x + 6 = x^2 - 2x + 1$. This equation can be rewritten as a quadratic equation in standard form: $x^2 - 4x - 5 = 0$. One way to solve this quadratic equation is to factor the expression $x^2 - 4x - 5$ by identifying two numbers with a sum of –4 and a product of –5. These numbers are –5 and 1. So the quadratic equation can be factored as $(x - 5)(x + 1) = 0$. It follows that 5 and –1 are the solutions to the quadratic equation. However, the solutions must be verified by checking whether 5 and –1 satisfy the original equation, $\sqrt{2x+6} + 4 = x + 3$. When $x = -1$, the original equation gives $\sqrt{2(-1)+6} + 4 = (-1) + 3$, or $6 = 2$, which is false. Therefore, –1 does not satisfy the original equation. When $x = 5$, the original equation gives $\sqrt{2(5)+6} + 4 = 5 + 3$, or $8 = 8$, which is true. Therefore, $x = 5$ is the only solution to the original equation, and so the solution set is $\{5\}$.

Choices A, C, and D are incorrect because each of these sets contains at least one value that results in a false statement when substituted into the given equation. For instance, in choice D, when 0 is substituted for x into the given equation, the result is $\sqrt{2(0)+6} + 4 = (0) + 3$, or $\sqrt{6} + 4 = 3$. This is not a true statement, so 0 is not a solution to the given equation.

QUESTION 8

Choice D is correct. Since $x^3 - 9x = x(x+3)(x-3)$ and $x^2 - 2x - 3 = (x+1)(x-3)$, the fraction $\frac{f(x)}{g(x)}$ can be written as $\frac{x(x+3)(x-3)}{(x+1)(x-3)}$. It is given that $x > 3$, so the common factor $x - 3$ is not equal to 0. Therefore, the fraction can be further simplified to $\frac{x(x+3)}{x+1}$.

Choice A is incorrect. The expression $\frac{1}{x+1}$ is not equivalent to $\frac{f(x)}{g(x)}$ because at $x = 0$, $\frac{1}{x+1}$ as a value of 1 and $\frac{f(x)}{g(x)}$ has a value of 0.

Choice B is incorrect and results from omitting the factor x in the factorization of $f(x)$. Choice C is incorrect and may result from incorrectly factoring $g(x)$ as $(x + 1)(x + 3)$ instead of $(x + 1)(x - 3)$.

QUESTION 9

Choice A is correct. The standard form for the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) are the coordinates of the center and r is the length of the radius. According to the given equation, the center of the circle is $(6, -5)$. Let (x_1, y_1) represent the coordinates of point Q. Since point P $(10, -5)$ and point Q (x_1, y_1) are the endpoints of a diameter of the circle, the center $(6, -5)$ lies on the diameter, halfway between P and Q. Therefore, the following relationships hold: $\frac{x_1 + 10}{2} = 6$ and $\frac{y_1 + (-5)}{2} = -5$. Solving the equations for x_1 and y_1, respectively, yields $x_1 = 2$ and $y_1 = -5$. Therefore, the coordinates of point Q are $(2, -5)$.

Alternate approach: Since point P $(10, -5)$ on the circle and the center of the circle $(6, -5)$ have the same y-coordinate, it follows that the radius of the circle is $10 - 6 = 4$. In addition, the opposite end of the diameter $\overline{PQ}$ must have the same y-coordinate as P and be 4 units away from the center. Hence, the coordinates of point Q must be $(2, -5)$.

Choices B and D are incorrect because the points given in these choices lie on a diameter that is perpendicular to the diameter $\overline{PQ}$. If either of these points were point Q, then $\overline{PQ}$ would not be the diameter of the circle. Choice C is incorrect because $(6, -5)$ is the center of the circle and does not lie on the circle.

QUESTION 10

Choice C is correct. Let x represent the number of 2-person tents and let y represent the number of 4-person tents. It is given that the total number of tents was 60 and the total number of people in the group was 202. This situation can be expressed as a system of two equations, $x + y = 60$ and $2x + 4y = 202$. The first equation can be rewritten as $y = -x + 60$. Substituting $-x + 60$ for y in the equation $2x + 4y = 202$ yields $2x + 4(-x + 60) = 202$. Distributing and combining like terms gives $-2x + 240 = 202$. Subtracting 240 from both sides of $-2x + 240 = 202$ and then dividing both sides by -2 gives $x = 19$. Therefore, the number of 2-person tents is 19.

Alternate approach: If each of the 60 tents held 4 people, the total number of people that could be accommodated in tents would be 240. However, the actual number of people who slept in tents was 202. The difference of 38 accounts for the 2-person tents. Since each of these tents holds 2 people fewer than a 4-person tent, $\frac{38}{2} = 19$ gives the number of 2-person tents.

Choice A is incorrect. This choice may result from assuming exactly half of the tents hold 2 people. If that were true, then the total number of people who slept in tents would be $2(30) + 4(30) = 180$; however, the total number of people who slept in tents was 202, not 180. Choice B is incorrect. If 20 tents were 2-person tents, then the remaining 40 tents would be 4-person tents. Since all the tents were filled to capacity, the total number of people who slept in tents would be $2(20) + 4(40) = 40 + 160 = 200$; however, the total number of people who slept in tents was 202, not 200. Choice D is incorrect. If 18 tents were 2-person tents, then the remaining 42 tents would be 4-person tents. Since all the tents were filled to capacity, the total number of people who slept in tents would be $2(18) + 4(42) = 36 + 168 = 204$; however, the total number of people who slept in tents was 202, not 204.

QUESTION 11

Choice B is correct. The x-coordinates of the x-intercepts of the graph are -3, 0, and 2. This means that if $y = f(x)$ is the equation of the graph, where f is a polynomial function, then $(x + 3)$, x, and $(x - 2)$ are factors of f. Of the choices given, A and B have the correct factors. However, in choice A, x is raised to the first power, and in choice B, x is raised to the second power. At $x = 0$, the graph touches the x-axis but doesn't cross it. This means that x, as a factor of f, is raised to an even power. If x were raised to an odd power, then the graph would cross the x-axis. Alternatively, in choice A, f is a third-degree polynomial, and in choice B, f is a fourth-degree polynomial. The y-coordinates of points on the graph become large and positive as x becomes large and negative; this is consistent with a fourth-degree polynomial, but not with a third-degree polynomial. Therefore, of the choices given, only choice B could be the equation of the graph.

Choice A is incorrect. The graph of the equation in this answer choice has the correct factors. However, at $x = 0$ the graph of the equation in this choice crosses the x-axis; the graph shown touches the x-axis but doesn't cross it. Choices C and D are incorrect and are likely the result of misinterpreting the relationship between the x-intercepts of a graph of a polynomial function and the factors of the polynomial expression.

QUESTION 12

Choice D is correct. Dividing both sides of equation $\frac{2a}{b} = \frac{1}{2}$ by 2 gives $\frac{a}{b} = \frac{1}{4}$. Taking the reciprocal of both sides yields $\frac{b}{a} = 4$.

Choice A is incorrect. This is the value of $\frac{a}{2b}$, not $\frac{b}{a}$. Choice B is incorrect. This is the value of $\frac{a}{b}$, not $\frac{b}{a}$. Choice C is incorrect. This is the value of $\frac{b}{2a}$, not $\frac{b}{a}$.

QUESTION 13

Choice C is correct. It is assumed that the oil and gas production decreased at a constant rate. Therefore, the function f that best models the production t years after the year 2000 can be written as a linear function, $f(t) = mt + b$, where m is the rate of change of the oil and gas production and b is the oil and gas production, in millions of barrels, in the year 2000. Since there were 4 million barrels of oil and gas produced in 2000, $b = 4$. The rate of change, m, can be calculated as $\frac{4 - 1.9}{0 - 13} = -\frac{2.1}{13}$, which is equivalent to $-\frac{21}{130}$, the rate of change in choice C.

Choices A and B are incorrect because each of these functions has a positive rate of change. Since the oil and gas production decreased over time, the rate of change must be negative. Choice D is incorrect. This model may result from misinterpreting 1.9 million barrels as the amount by which the production decreased.

QUESTION 14

Choice C is correct. The second equation of the system can be rewritten as $y = 5x - 8$. Substituting $5x - 8$ for y in the first equation gives $5x - 8 = x^2 + 3x - 7$. This equation can be solved as shown below:

$$x^2 + 3x - 7 - 5x + 8 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

$$x = 1$$

Substituting 1 for x in the equation $y = 5x - 8$ gives $y = -3$. Therefore, $(1, -3)$ is the only solution to the system of equations.

Choice A is incorrect. In the xy-plane, a parabola and a line can intersect at no more than two points. Since the graph of the first equation is a parabola and the graph of the second equation is a line, the system cannot have more than 2 solutions. Choice B is incorrect. There is a single ordered pair (x, y) that satisfies both equations of the system. Choice D is incorrect because the ordered pair $(1, -3)$ satisfies both equations of the system.

QUESTION 15

Choice D is correct. Since $h(x) = 1 - g(x)$, substituting 0 for x yields $h(0) = 1 - g(0)$. Evaluating $g(0)$ gives $g(0) = 2(0) - 1 = -1$. Therefore, $h(0) = 1 - (-1) = 2$.

Choice A is incorrect. This choice may result from an arithmetic error. Choice B is incorrect. This choice may result from incorrectly evaluating $g(0)$ to be 1. Choice C is incorrect. This choice may result from evaluating $1 - 0$ instead of $1 - g(0)$.

QUESTION 16

The correct answer is 3. The solution to the given equation can be found by factoring the quadratic expression. The factors can be determined by finding two numbers with a sum of 1 and a product of -12. The two numbers that meet these constraints are 4 and -3. Therefore, the given equation can be rewritten as $(x + 4)(x - 3) = 0$. It follows that the solutions to the equation are $x = -4$ or $x = 3$. Since it is given that $a > 0$, a must equal 3.

QUESTION 17

The correct answer is 32. The sum of the given expressions is $(-2x^2 + x + 31) + (3x^2 + 7x - 8)$. Combining like terms yields $x^2 + 8x + 23$. Based on the form of the given equation, $a = 1$, $b = 8$, and $c = 23$. Therefore, $a + b + c = 32$.

Alternate approach: Because $a + b + c$ is the value of $ax^2 + bx + c$ when $x = 1$, it is possible to first make that substitution into each polynomial before adding them. When $x = 1$, the first polynomial is equal to $-2 + 1 + 31 = 30$ and the second polynomial is equal to $3 + 7 - 8 = 2$. The sum of 30 and 2 is 32.

QUESTION 18

The correct answer is $\frac{3}{2}$. One method for solving the system of equations for y is to add corresponding sides of the two equations. Adding the left-hand sides gives $(-x + y) + (x + 3y)$, or $4y$. Adding the right-hand sides yields $-3.5 + 9.5 = 6$. It follows that $4y = 6$. Finally, dividing both sides of $4y = 6$ by 4 yields $y = \frac{6}{4}$ or $\frac{3}{2}$. Any of 3/2, 6/4, 9/6, 12/8 or the decimal equivalent 1.5 will be scored as correct.

QUESTION 19

The correct answer is 8. The number of employees, y, expected to be employed by the company x quarters after the company opened can be modeled by the equation $y = ax + b$, where a represents the constant rate of change in the number of employees each quarter and b represents the number of employees with which the company opened. The company's growth plan assumes that 2 employees will be hired each quarter, so $a = 2$. The number of employees the company opened with was 8, so $b = 8$.

QUESTION 20

The correct answer is 144. In a circle, the ratio of the length of a given arc to the circle's circumference is equal to the ratio of the measure of the arc, in degrees, to 360°. The ratio between the arc length and the circle's circumference is given as $\frac{2}{5}$. It follows that $\frac{2}{5} = \frac{x}{360}$. Solving this proportion for x gives $x = 144$.