Commentary

Neptune, I

1. **(89)** Add the four scores, then divide by the number of scores.

2. **(46¢ each)** Begin with the $2 Stella gave the clerk, subtract 16¢. Since she bought 4 pencils, divide by 4. (They must have been mechanical pencils!!)

3. **(School and studying, t.v.)** The only two activities that equal half his time are "school" and "studying." The activity that takes twice as much time as studying would have to take 20% of his time. The only activity that takes 20% of his time is "T.V."

4. **(between 5:30 and 5:45)** Since there are only 176 seventh graders, at some point between 5:30 and 5:45 student number 176 was notified, and the remainder of the students called had already been notified by someone earlier. The work below shows this analysis:

Russell finds out at 4:00 P.M.	one 7th grader knows
Russell tells 2 others at 4:15 P.M.	1+2 = 3 (7th graders that know)
2 each tell 2 more at 4:30 P.M.	3+4 = 7 (7th graders that know)
4 each tell 2 more at 4:45 P.M.	7+8 = 15 (7th graders that know)
8 each tell 2 more at 5:00 P.M.	15+16 = 31 (7th graders that know)
16 each tell 2 more at 5:15 P.M.	31+32 = 63 (7th graders that know)
32 each tell 2 more at 5:30 P.M.	63+64 = 127 (7th graders that know)
64 each tell 2 more at 5:45 P.M.	127+128 = 255

5. **(5, 12)** Trial and error would be one way to solve this problem. Try multiplying numbers together that differ by 7. $0 \times 7 = 0$; $1 \times 8 = 8$; $2 \times 9 = 18$; $3 \times 10 = 30$; $4 \times 11 = 44$; $5 \times 12 = 60$ Another method would be to assign the first unknown a value of X and the second variable the value of $X + 7$. You could then solve the equation $X(X + 7) = 60$, to find $X = 5$, $X + 7 = 12$.

6. **(15)** Students will find it helpful to make a list and keep track of combinations. These include one digit only, two-digit combinations and three-digit combinations.

1	1-2	3-1	1-2-3	3-1-2
2	2-1	2-3	2-3-1	2-1-3
3	1-3	3-2	3-2-1	1-3-2 (For a total of 15 combinations.)

7. **(56)** One way would be to start at one corner of the rectangle and place a dot (fence post) every five feet. If the dots are counted, they will find that 56 fence posts are needed.

8. **(36' by 44')** The number of 1/2" segments in 18" is 36 so the yard is 36' wide; the number of 1/2" segments in 22" is 44 so the yard is 44' long.

9. **(-13 oz)** The television by itself is 40 oz; with the balloon it's 27 oz. Therefore the balloon must counterbalance 13 oz. Since weight is considered positive, something that counterbalances it would be assigned a negative value. So the helium balloon would be assigned a weight of -13.

Commentary

Neptune, II

1. (8) An organized list would be helpful, such as: 50, 25 + 25, 25 + 10 + 10 + 5, 25 + 10 + 5 + 5 + 5, 25 + 5 + 5 + 5 + 5 + 5, 10 + 10 + 10 + 10 + 10, 10 + 10 + 10 + 10 + 5 + 5, 10 + 10 + 10 + 5 + 5 + 5 + 5. The total number of ways is 8.

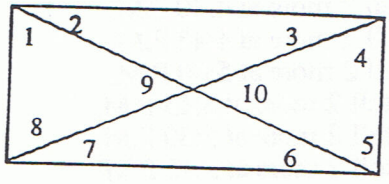
2. **(16)** *Guess-check-revise* would be a good approach to this problem. Start by guessing a number for x, and check against the conditions. Sixteen works: $(16 - 4) + (16 + 8) = 36$

3. **(20 cups)** 2 quarts+$1\frac{1}{2}$ quarts+ $\frac{1}{2}$ quarts=5 quarts. Since 4 cups=1 quart,4 x 5=20 cups.

4. **(20, 21)** One pattern begins with 5, then adds 1, then 2, then 1, then 2 alternating. The next number in the pattern would add 2, resulting in 20, then add 1, resulting in 21. Another solution is to notice that these are simply the counting numbers, starting with 5, leaving out every 3rd number.

5. **(2, 3)** The notion is to extend the pattern above in the other direction, by reversing the action.

6. **(10)** Draw a rectangle with the two diagonals. Count the acute (less than 90 degrees) angles.



7. **(14, 12, 13, 11).** One good approach is to make a table, and proceed by eliminating possibilities. From the first clue, we know David isn't 11 and isn't 14, so we place X marks in the chart. That same clue tells us Kelly isn't 14 and Lisa isn't 11. After the first clue, the chart looks like:

	11	12	13	14
Lisa	X			
Drew				
David	X			X
Kelly				X

Another approach is just to line up names one on top of the other, and manipulate them according to the rules until you happen upon the solution.

8. **(91 days)** Make a table for the different combinations. For example: pepperoni can be combined with each of the other 13 items, sausage can be combined with the other 12 items, meatball can be combined with the other 11 items and so on. Total the different combinations , $13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 91$

9. **(about 4.6 or 4 1/2 years)** Divide the $2,000,000 by $50 to figure how many times you will give away $50. You need to give away money for 40,000 hours. Divide the 40,000 hours by 24 (the hours in a day) and then by 365 (the days in a year) to get an answer of about 4.6 years.

Commentary

Neptune, III

1. (13) Organize the information. There are 9 small triangles, 3 medium triangles made from four small ones, and one large triangle.

2. (**40 - 6 x (6 - 2) - 6 = 10**) Students will have to recall the order of operations (multiply and divide first then add and subtract). The solution is then easily found.

3. **(21,000,000)** The total number of calls that can be handled is 25 x 700,000 or 17,500,000. Only 5% will be inaccurate. Compute 5% of 17,500,000 which equals 875,000 per hour. This number is multiplied by 24 for the number of calls per day, mishandled.

4. **($3)** One way to quickly figure the tip of 20% is to find 10% of $15, which is $1.50, and then realze that 20% would be twice as much, or $3. Another way is that most people know intuitively that 2 x 15 is 30, so .2 x 15 is 3.0, meaning $3. Yet another way is that most people realize that 20% of $10 would be $2, so $2 plus half of $2, or $2 + $1, is the tip for $15.

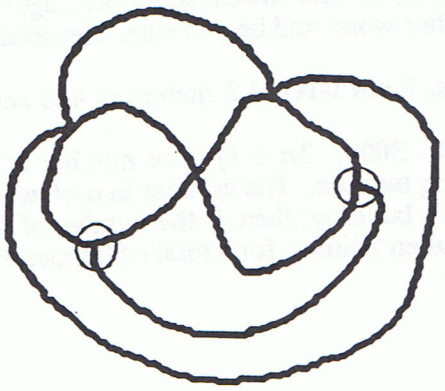
5. **($9)** Estimate spaghetti at about $1 a pound so 2 pounds would cost $2; sauce at about $2.50 a jar or 2 jars for $5; bread is about 70¢ a loaf so three loaves would be about $2. The total to the nearest dollar is $9.

6. **(340, 71)** The two sides that are 10 by 8 have an area of 80 each for a total of 160. Likewise, the two 8 by 5 and the two 10 by 5 sides have total areas of 80 and 100, respectively. The sum of the six sides gives the amount of contact paper needed. The ribbon needed would be two 10 inch pieces, two 8 inch pieces, four 5 inch pieces, and 15 inches for the bow. This totals 71 inches.

7. **(94°)** The difference between 88° F and -6° F is 94°. So it is warmer by 94 degrees in Miami.

8. **(37¢)** If notebooks cost $1.77 for two, then 6 cost $5.31. Multiply $5.31 by 1.06 to find the cost after sales tax has been added, which is $5.63. Subtract $5.63 from $6.00 to find the change.

9. The two places where one can start and trace this path, without lifting their pencil, are circled below. This is because these are the only two places with an *odd number* of paths coming into or leaving out of them. So you can start at either place, and you'll finish at the other place.

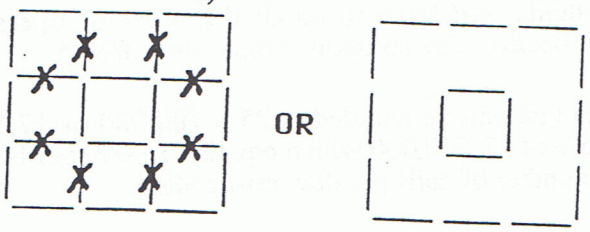


Commentary

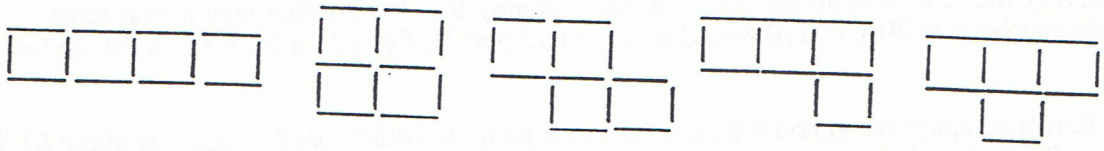
Neptune, IV

1. **(1 hour, 5 minutes)** The difference in 4:10 PM and 6:00 PM is 1 hour and 50 minutes. He spends 45 minutes driving, leaving 1 hour and 5 minutes at the library.

2. **(Answer below.)**



3. **(5)** The 5 possible configurations are below:



4. **(11)** For a first time customer the first 6 games are free. Then you can play 4 more times, costing $3.00, but get a free game. So the total for only $3.00 is 11 games.

5. **(11 or 12)** 3 apples for $1.29 means that each apple costs about $1.29 ÷ 3, or 43¢ each. Then $5.00 ÷ 43¢ is 11.6, so about 11 or 12 apples, on average, can be purchased for $5.

6. **(35)** Work backwards: 372 ÷ 6 = 62, and 62 - 27 = 35. So $x = 35$.

7. **(9.0, 8.3, 9.3, 8.43; bars; vault; 35.03)** To find the mean for each event, add the respective numbers:

floor: 8.8; 9.3; and 8.9
bars: 8.2; 8.2; and 8.5
vault: 9.4; 9.5; and 9.0
beam: 8.4; 8.4; and 8.5

and divide by 3. Her lowest score (average) is 8.3 for bars, the highest is 9.3 for vault, so those are her worst and best events. Her total is the sum of all four individual event scores.

8. **(19 pieces, 2 cm left)** 4.2 meters is 420 centimeters. 420 ÷ 22 = 19, r 20.

9. **(101; 500; 3001; $3n + 1$)** The number of blocks in the towers is always one more than the building number. The number in each wing is the same as the building number. The total blocks for a building, then, is the number of the building plus 1, plus the number of the building taken 2 times, for a total of 3 times the building number, plus 1.

Commentary

Neptune, V

1. **(0.6)** 30% of 100 is 30, and 20% of 30 is 6. Then 10% of 6 is 1/10 of 6, or 0.6. Or, change the percents to decimals and solve the mathematical sentence:
   0.1 x 0.2 x 0.3 x 100 = 0.6

2. **(216)** If the hundreds digit is 1/3 the ones digit, then the ones digit can only be multiples of 3 -- 3, 6, or 9. This means the hundreds digit can only be 1, 2, or 3. The hundreds digit is also two times the tens digit. The only hundreds digit of that group that is twice another whole number is two. If the hundreds digit is two, the tens digit must be one and the ones digit must be six.

3. **(36)** There are 12 cubes in each level and three levels for a total of 36 cubes. Students may know that volume = length times width times height. *3 x 4 x 3 = 36 cubic units*

4. **(2)** The only cubes without paint would be the two single cubes in the center. To see this, each of the 12 cubes in the top and bottom layer have paint on them. In the middle layer, the 3 on each end, and the 2 on each side, in the center of the side, would have paint. This leaves only 2 out of the layer of 12 with no paint.

5. **(8)** Drawing a diagram will help students solve this problem. A vertical line best represents the well. Many students will realize that the snail is making progress at $2\frac{1}{2}$ feet per 24-hour period, and can simply start at $2\frac{1}{2}$ and count by $2\frac{1}{2}$ until they get to 20.

6. **(375)** The problem has students find the difference between 345 and -30 on a number line.

7. **($\frac{10}{48}$ or $\frac{5}{24}$ or 21%)** One approach is to *guess-check-revise*, knowing that once you chose a number for the red marbles, the other marbles can be obtained and you can check to see if they total 48. If not, revise the guess for the red marbles, and check again.

   The solution can be obtained algebraically by letting X be the number of red, $2X$ the number of yellow, $X + 6$ the number of white, and $2X + 12$ the number of blue marbles. The sum of the 4 colors is 48 so $X + 2X + X + 6 + 2X + 12 = 48$. Simplifying and solving for X gives $X = 5$. So there are 5 red, 10 yellow, 11 white, 22 blue for a total of 48 marbles. The probability of yellow is then 10/48.

8. **(+)** 7043+84 = 7127

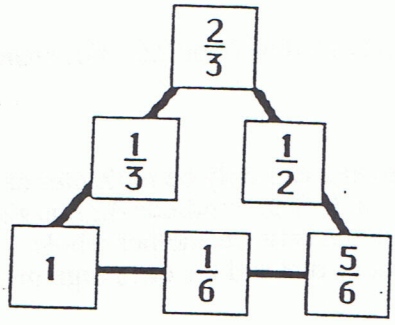
9. **(3)** This problem will later be solved as a system of equations. At this point, students will reason in a variety of ways to find the answer. One such way is this:

   > Use the second scale. Double what's on each side, and 2 jars of paste balance 2 scissors and 2 pencils. Substitute 2 scissors and 2 pencils for 2 jars on the left side of the first scale; 2 scissors and 2 pencils and another scissors then balance 8 pencils. Remove 2 pencils from both sides, then 3 scissors balance 6 pencils, or 1 scissors balances 2 pencils. Substituting 2 pencils for the scissors on the right side of the second scale means a jar of paste balances 3 pencils.

Commentary

Neptune, VI

1.



2. **(6)** The question is what two numbers add to 15, and one is three more than the other. Nine and six fit that description, so the team won 9 and lost 6 games.

3. **(8,999,999,991)** The way most students will find the product is to know that they must count down 5 more problems like these 3, and fill-in the patterns going down. The first digits are just the counting numbers 1, 2, 3, ..., so 5 more after that would be 8. The middle part of the answer would be 8 nines, consistent with the pattern. The last digit follows the pattern of counting down from 8, resulting in 1. The product is then predicted to be 8,999,999,991. Notice that it hasn't been *proved* that this is the product -- you've simply made a prediction.

4. **(3)** If 3 math students do 3 math problems in 3 minutes then 1 math student does 1 math problem in 3 minutes. If 33 students are doing 33 problems then each student is doing 1 problem. We know it takes a student 3 minutes to do 1 problem. Therefore, 33 students can do 33 problems in 3 minutes.

5. **(5)** Find the least common multiple of 10 and 8 which is 40. If each pizza has 8 slices and 40 slices are needed then $40 \div 8 = 5$ pizzas, allowing each person exactly 4 slices. Or a student might simply ask what is the fewest number of pizzas in which the total number of slices is divisible by ten. One pizza has 8 slices, 2 have 16, 3 have 24, 4 have 32, and 5 pizzas have 40 slices. Since 40 is the first number divisible by 10, five pizzas is the answer.

6. **(x)** $934 \times 463 = 432,442$

7. **(3, 4, 5)** Use mental math to find the sum and product of three consecutive numbers. Keep track of your results until you find that $3 + 4 + 5 = 12$ and $3 \times 4 \times 5 = 60$. The sum is 1/5 of the product.

8. **(31)** Work backwards. She ended with 1/4 of a dozen, or 3 donuts. If she gave her sister 1/2 of her donuts and half a donut, she had 7. (1/2 of 7 less 1/2 = 3) If she gave her brother 1/2 of her donuts plus 1/2 of a donut she must have had 15. (1/2 of 15 less 1/2 = 7) If she gave her mother 1/2 of her donuts plus 1/2 of a donut she must have had 31 to start. (1/2 of 31 less 1/2 = 15)

9. **(a. Sasha, Roberto, Lu; b. Sasha, Lu; c. somewhere from 1:30 to 1:45; d. Roberto)** The problem involves reading a graph over time, that tells a story. The order of finishing is the one who reached 1600 meters first, timewise. All three stopped at the graph height corresponding to 1600 m; but Sasha's line is to the left of the others, indicating she took less time. Sasha started the slowest because her graph at the beginning doesn't rise as fast as the other two -- she doesn't cover as many meters for any given time as the others, to start. Sasha did pass Lu, however, at about 1 and 1/2 to 1 and 3/4 minutes. Roberto ran the same pace all the way through, because his line goes in a constant direction -- he covers about 400 m each minute.

Commentary

Neptune, VII

1. $\left(\frac{325}{999}\right)$ Let n be the number $0.\overline{325}$, and then $1000n$ "moves the decimal" three places to the right, so $1000n$ is $325.\overline{325}$ ... Notice when you then subtract n from $1000n$, you get $999n = 325$. Therefore n is $325/999$.

2. **(15, 17, 19, 21, 23)** This problem can be approached through *guess-check-revise*; another method is to notice that since $95 \div 5 = 19$, the 5 consecutive odd numbers will be centered on 19.

   The problem can also be approached algebraically. The first odd number is x; the next consecutive odd numbers are $x + 2$, $x + 4$, $x + 6$, and $x + 8$. They total 95, so $x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 95$. Simplifying and solving the equation gives $5x + 20 = 95$, or $5x = 75$, so $x = 15$. The other numbers are the next four odd numbers in sequence.

3. **($3)** Work backwards: 50¢ plus the 25¢ lost gives 75¢. She had spent half of what she had and was left with 75¢, so prior to this step she had $1.50. If she gave half of what she had to her brother and was left with $1.50, then prior to this step she must have had $3.00.

4. One solution is shown below. Other solutions are possible.

4	9	2
8	1	6
3	5	7

5. **(1973)** Since all of the years from 1970 to 1980 have the thousands and hundreds digits the same, and their sum is 10, the other two digits must sum to ten. Therefore since a 7 has to be in the tens place to be in the 1970s decade, the units digit is a 3.

6. **(7)** 75% of 30 is 0.75 x 30 or 22.5. If he misses 8, his score would be 22 which is less than 75% of 30. So he must miss no more than 7.

7. **(144 cm^2)** The hexagon has 6 sides; $72 \div 6 = 12$ cm per side of the hexagon and therefore the square. The area of the square is side x side or 12 cm x 12 cm which equals 144 cm^2.

8. **(See below.)** The digits in the addend on the left can be reversed top and bottom, but the answer is unique. Both the answer and the individual numbers are unique in the subtraction problem.

$$\begin{array}{r} 97531 \\ \underline{86420} \\ 183951 \end{array} \qquad \begin{array}{r} 50123 \\ \underline{-\ 49876} \\ 247 \end{array}$$

9. **(a. $y + 13.7 = 78.69$ and $y = 64.99$; b. $4x + 3.9 = 47.9$ and $x = 11$; c. $n - 25 = 50$ and $n = 75$)** The equations result from using a variable y, x, or n to stand for the unknown weight on the scale. In A, a book of unknown weight y, plus 13.7, weighs 78.69. When the weight 13.7 is removed, the scale would show 78.79 - 13.7, or 64.99. In B, four books and a weight of 3.9 weigh 47.9. When the 3.9 is removed, the four books alone would weigh 47.9 - 3.9, or 44. If four books weigh 44, then each one must weigh $44 \div 4$ or 11. In C, a helium balloon pulls up on the scale and so has negative weight. Therefore when the helium balloon is removed from the scale, the scale would show 50 + 25 or 75. So the book weighs 75.
   [Note: Some students may write $x + x + x + x + 3.9 = 47.9$ for B, which is acceptable.]

Commentary

Neptune, VIII

1. **(20 chickens, 16 cows)** Most students will solve this by guess-check-revise. The number of chickens plus the number of cows has to be 36 since there are 36 heads. So students can guess numbers of chickens and cows that total 36, compute the number of legs, and adjust their guess accordingly until they reach the correct number.
   Another clever approach that some students will use is to reason that since there are at least 2 legs on each of the 36 heads, 72 legs are accounted for already. The extras, 104 - 72 or 32, must be to make 4-legged animals. That would be 16 pairs of extra legs, so there must be 16 cows. Then the rest, 36 -16 = 20, are chickens and have only the two legs given initially.

2. **(A, D)** A closed cube must have 6 faces. E has only 5 faces and cannot be a closed cube. B and C will not fold to make a cube, but A and D will.

3. **(150°)** At 5 o'clock, one hand is on the 12 and the other is on the 5. The portion of the clock covered by this angle is 5/12 of the clock face, which is a circle of 360°. (5/12) x 360° = 150°.

4. **(a. 12/38 or 6/19 or 32%; b. 11/37 or 30%)** On your first pull, you have 12 chances out of 38 of pulling a white sock. If a white sock is pulled first, then there are only 11 white socks left out of a total of 37 left. The chance is then 11/37.

5. **(21; 28; 36)** The most obvious pattern begins with one and first adds two, then adds three, then adds four, then adds five. This pattern is continued by adding six, then seven, then eight. The next three numbers would be 15 + 6 or 21, 21 + 7 or 28, 28 + 8 or 36. Another way to find the answers is to notice that the *nth* term is given by $n(n + 1) \div 2$. If students can justify other answers because they noticed other patterns, give them credit.

6. **(85%)** The average is the total of all the scores divided by the number of tests. The highest possible average would be achieved by scoring a 100% on test number four. The sum would then be 65 + 90 + 85 + 100 = 340. The highest possible average would be 340/4 = 85%.

7. **(6)** The dogs are eating simultaneously, so it takes each dog 2 minutes to eat 2 bones; that is one bone per minute per dog. So four dogs can eat six bones each, a total of 24 bones, in six minutes .

8. **(28 - (20 - 3) - 4 = 7)**

9. **(back, left, right)** Students who can solve this easily have a good sense of space and their visualization skills are high. The visual clues they will use are the figures which are in front of or behind others, or to the left and right of others.
   In the left most picture, the cola is now in front of the other two, whereas it was behind them in the given picture. So this picture is the reverse of the given picture, meaning the view is from the back of the counter. In the center picture, the fries are hidden by the burger -- this would happen only if the view was from the left. In the last picture, the fries are visible in front of the burger -- this implies a view from the right of the original picture. Students might want to experiment with a similar situation, using 3 real objects.

Commentary

Neptune, IX

1. **(10)** This problem requires visualizing the blocks that are hidden. In the back left, the blocks are stacked 3 high; back right, front left and front right are stacked two high; there is a single block in front.

2. a) You would give the clerk a penny to get fewer coins back in change.

   b) **($5.25)** Your change would be a five-dollar bill and a quarter.

3. **($2.00 to $2.50)** One way to estimate 15% is to estimate 10% of a number, then add half of that for the other 5%. 10% of $15.32 would be about $1.50, and adding half of that would be another 75¢. The total would be about $2.25. Accept an estimate between $2.00 and $2.50.

4. **(12)** One way to approach the problem is to choose a day to begin counting, and cycle through every 3 days, every 4 days, and every 6 days from that point. Eventually all three counts include a common day, which is 12 days from when you start. You could also solve the problem by finding the least common multiple of 3, 4, and 6, which is 12.

5. **(50%)** The rectangle is divided into six equal sections, three of which are shaded. This could be expressed as $\frac{3}{6}$, $\frac{1}{2}$, 0.5, or 50%.

6. **($17\frac{1}{2}$ hrs.)** A speed of 50 miles per hour means 100 miles every two hours, or 800 miles in 16 hours. Add another hour for the next 50 miles, and you're up to 17 hours for 850 miles. There's 20 more miles to go, which is close to 25 miles that would take almost a half hour at that speed. So add another half hour to the estimated time.

7. **(72)** The area of 5 squares is 180, so the area of one square is 180 ÷ 5 or 36 square units. If the area of one square is 36 square units, then each side must equal 6 units. There are 12 sides that make up the perimeter of the figure, and 12 x 6 = 72.

8. **(W=3; R=7; N=9; G=1; T=2; I=4)** Trial and error works, but careful analysis helps. Logic will tell you first that G=1 since the column with two zeros added results in another number, which can only be 1. With that start, the puzzle is not difficult to finish.

Commentary

Neptune, X

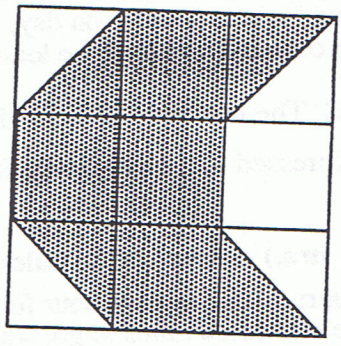
1. (25%) If 60 out of 80 teachers are female, then 20 are male. 20 males out of 80 total teachers equals 25%.

2. **(12)** For each of the four hats, a different bow will make a different hat. 3 different *green* hats + 3 different *yellow* hats + 3 different *blue* hats + 3 different *purple* hats = 12 different hats.

3. **($68.44)** If each sandwich feeds 8 people, he will need 4 sandwiches to feed himself and 30 friends. $15.99 x 4 = $63.96. Adding sales tax of 7% can be done in one step by multiplying $63.96 x 1.07 to get $68.44, when rounded up to the next cent.

4. **(26)** For her overall average to be 70% she must sink 35 out of 50 baskets because 35/50 = 0.70 = 70%. Therefore if she needs to sink a total of 35 baskets and has already sunk 9, she needs to sink 26 out of her next 30 attempts (35 - 9 = 26).

5. **(54 sq. in.)** There are a number of ways to find this answer. One approach is to determine that each of the nine smaller squares has area 81 ÷ 9 or 9, and count whole (4) and half squares (4) in the shaded part for a total area of 4 x 9 + 4 x 4.5 = 54. Another way, once you know each small square has area 9, is to count the whole (1) and half squares (4) in the unshaded part and subtract these amount from 81. Yet another way is to notice that moving the unshaded triangles on the left, with the unshaded figure on the right, produces an unshaded figure that is 1/3 of the total figure. So the shaded part would be 2/3 of 81, which is 54.



6. **(11)** Counting by 1.5 makes the full sequence:

   2.5, 4, 5.5, 7, 8.5, 10, 11.5, 13, 14.5, 16, 17.5 for a total of 11 terms.

7. **(14)** Students should be encouraged to start analyzing such figures in an organized way. One such approach for this problem is to start counting the smallest squares first, getting 9. Then move to the next smallest size, which is a 2-by-2 square – there are 4 of these. The next size square is the large one itself, giving a total number of 9 + 4 + 1.

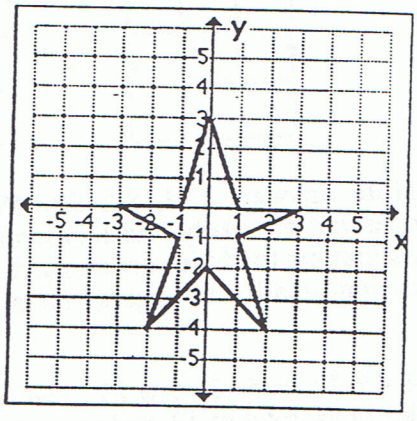
8. **($77)** They will need $152 for 4 admissions to Disney World. Since they already have $75, they will need to save $77 more.

Commentary

Neptune, XI

1. **(6)** Label the cars, except for the engine, A, B, and C. Then the different arrangements are ABC, ACB, BAC, BCA, CAB, and CBA.

2. **(The figure is shown to the right.)**



3. **(Point E)** Point A is 1/4 and B is 1/2. Then 1/4 of 1/2, or 1/8, is point E.

4. **(Monday)** 100 days ago was 14 weeks and 2 days ago. If today is Wednesday, 14 weeks ago was also Wednesday and 2 days before that was Monday.

5. **(5)** Students might use a chart like this:

BUILT	NOT BUILT	EARNINGS
1	9	-50
2	8	0
3	7	50
4	6	100
5	5	150

Another approach would be algebraic, setting up and solving the equation $40x - 10(10 - x) = 150$, where x is the number of dog houses made.

6. **(2:00 P.M.)** 4 hours + 15 minutes + 30 minutes + 20 minutes = 5 hours and 5 minutes after 9 A.M. means Mary will return home at 2:05 P.M. Since the question asks "about what time," 2:00 P. M. is a reasonable answer.

7. **(86)** $112 + 88 + 80 + 65 = 345$, and $345 \div 4 = 86.25$ This answer is rounded to the nearest whole number, 86.

8. **(17 grams)** Students can reason that the only difference between the two pictures is that 2 more balloons have been attached to one scale, resulting in 6 less grams. This implies that a balloon weighs -3 grams. Using this fact and the first scale, we have that 2 cups of coffee and -3 grams equals 31 grams, so 2 cups of coffee alone would be 3 more grams than 31, or 34 grams. Therefore one cup of coffee is half that amount, or 17 grams.

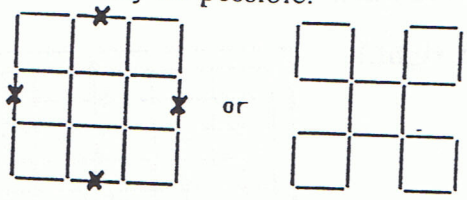
9. **($6t + 15.7 = 58.3$, $t = 7.1$)** Students might write a different form of the equation, such as $t + t + t + t + t + t + 15.7 = 58.3$, which is acceptable.

Commentary

Neptune, XII

1. (Sample: $4 + 4 + 4 \div 4 = 9$)

2. **(One correct answer would be to remove the outside middle segments on each of the 4 sides.)** Others may be possible.



3. **(One possible answer: {70, 80, 90, 100} has a mean of 85 and a median of 85.)** Other answers are likely. Each answer will have to be checked individually. The mean is found by summing the four numbers and dividing by four. The median is found by ordering the numbers from smallest to largest, and taking the mid-point between the two middle numbers.

4. **(15)** $5 + 8 + 4 = 17$ and $17 - 2 = 15$. The two students who are on two teams have been counted twice in the total of 17, and so must be removed from the count one time.

5. **(45)** Person 1 shakes hands with 9 others, person 2 shakes hands with 8 others, and so on. Another way to see this is with a diagram, as started to the right. A line would be drawn to connect all ten points. The total number of lines is $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$.

6. **(3/42 or 1/14 or about 7%)** The chances you'll scratch off an even number the first time is 4/8 or 1/2. If you succeed, the chances you'll scratch an even number is 3/7, since there are now three even numbers left out of the seven spots. If successful, the chances you'll scratch off an even number the third time in a row is 2/6 or 1/3. The chances you'll succeed on all three scratches is then the product of 1/2, 3/7, and 1/3. This computation results in 3/42 or 1/14 or 0.07 when rounded, which is about 7%.

7. **($4)** The area of the small cookie is πr^2, where r is the radius of the small cookie. The giant cookie has 4 times the diameter, and hence 4 times the radius, of the small cookie, so it's area is $\pi(4r)^2$ or $16\pi r^2$. Therefore the new cookie has 16 times the area of the small cookie, and should cost 16 x 25¢ or $4.

8. **(55, 385, 13th)** The pattern that students will discover for making the buildings is to add the square of the building number to make the next building in line. For example, the number of cubes for the third building is $9 + 4 + 1$. The number for the 4th building can be found by adding on to this building a 4-by-4 bottom layer, or 16 more blocks. The 5th building could then be made by adding a 5-by-5 bottom layer, increasing the number of blocks by 25. The 10th building would then take $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$, or 385, blocks. If you continue adding the squares of the next few whole numbers, the sum for building 13 is 819 and the sum for building 14 is 1015, so you could only make building 13 with 1000 blocks.

9. **(50)** If you pair each odd number from 1 to 99 with the next even number from 2 to 100, each odd number would be 1 less than its partner. There are 50 such pairs, so the difference is 50.

Commentary

Neptune, XIII

1. **(10 cubes)** 6 on bottom row, 3 in the middle, one on top.

2. **(4:20 P.M.)** Multiples of 16 are 16, 32, 48, 64, and 80. Multiples of 20 are 20, 40, 60 and 80. Therefore in 80 minutes after 3:00 P.M. both buses will arrive at the mall at the same time. (80 is the least common multiple of 16 and 20.)

3. **(6'2")** One approach is to convert the heights to inches. If the average height is 73" then the total team height is 5 x 73" = 365". Solve 73" + 75" + 72" + 71" + x = 365". The fifth player's height x is then 74" or 6'2" . Another approach is to look at the given heights and compare them to the given average -- the first height 6'1" is right on the average, so give it a 0. The 6'3" would be +2, the 6' would be -1, the 5'11" would be -2 from the given average. To get all of these integers back to zero, you need one that's +1. So the last player would have to be 1" more than the average, or 6'2".

4. **(84)** Various strategies may be used including trial and error. Since 12 + 7 = 19 and 12 - 7 = 5, the numbers are 12 and 7. Their product is 84.

5. **(about 5 1/2 months)** 1000 hours ÷ 6 hours per day is approximately 167 days. This is about 5 and one-half months.

6. **($180)** $45,000,000,000 ÷ 250,000,000 = $180.

7. **(a. 63%, b. 38%, c. 5%)** For (a), he will get on base if the dart lands in 5 out of the 8 equal areas, hence the percent is 5 ÷ 8 = 0.625 or about 63%. Likewise, he'll make an out 3 ÷ 8 = 0.375 or about 38% of the time. He would make 3 outs in a row about 38% x 38% x 38% of the time, or about 5% of the time.

8. **(150)** Compute $\frac{3}{5} \times 250 = 150$, or change $\frac{3}{5}$ to a decimal and compute 0.6 x 250 = 150. For some students, it might be easier to think of working with the ratio in an intuitive fashion.

   Thus, the ratio is:

   3 to 5
   6 to 10
   30 to 50
   .
   .
   .
   150 to 250

Commentary

Neptune, XIV

1. **(a. 168,000 - 170,000 b. 6 weeks)** This problem encourages students to use round numbers, since these are only estimates to begin with. They might begin with rounding 980 breaths per hour to 1000 breaths per hour, and multiply this by 24 hours/day and 7 days/week, to get 168,000. This amount could be rounded to 170,000. For (b), students can divide 1 million by 168,000 or 170,000 to get about 6 weeks. This answer might be written in equivalent forms, such as 1 1/2 months.

2. **(6)** 20 dimes and 30 pennies is \$2.30. If Ken has \$2.30 with 8 quarters and the rest nickles, then the 30¢ must be all nickles, or 6 nickles.

3. **(36)** If the perimeter is 48 cm, then each side is 48 ÷ 8 or 6 cm. Therefore the side of the square is 6 cm, which means its area is 36 sq. cm.

4. **(\$7.25)** \$4.00 + 6(\$1.75) = \$14.50. \$14.50 ÷ 2 = \$7.25

5. **(40 green, 60 red)** A possible solution is to "ratio up" the given amounts. He pulled out 25 jelly beans, and 10 were green. If he grabbed this same amount four times, he would have pulled out all 100 jelly beans, and have pulled 10 green ones each time, for a total of 40. There must then be 100 - 40 or 60 red jelly beans also.

6. **(35 feet)** Students might solve a proportion to find the length x. In similar rectangles, the sides are proportional, so $\frac{15}{6} = \frac{x}{14}$. Finding a common denominator so the numerators of the fractions can be compared, we have that $\frac{15 \cdot 14}{6 \cdot 14} = \frac{6 \cdot x}{6 \cdot 14}$. This implies the numerators are equal, or that $15 \cdot 14 = 6x$. Solving for x by dividing both sides by 6, we have that $x = 35$.

7. **(43)** 254 people divided by 6 people/row gives 42 r 2 rows needed. This means that 43 rows were actually needed to hold all 254 passengers.

8. **(22 times)** The hands coincide once every hour except between 11 and 1. Between 11:00 and 1:00 the hands coincide only once: 3:17, 4:22, 5:27, 6:33, 7:38, 8:43, 9:49, 10:54, 12:00, 1:06, 2:11. Each one happens twice; therefore 2 times 11 = 22 times.

Commentary

Neptune, XV

1. **(0)** A number sentence for the action is: 7 - 2 + 4 - 5 + 3 - 1 + 2 - 1 = 7. He had 7 pogs when he started, and 7 when he finished.

2. **(a. 19; b. answers will vary)** The first 8 letters cost 74¢, leaving $1.84 -$0.74 or $1.10 left for the other letters. At 10¢ each, this comes to 11 more letters. This gives 11 + 8 or 19 as the total. To check (b), make a chart, such as:

   8 or fewer ... 74¢
   9 letters 84¢
   10 letters 94¢ etc.

   Use this chart to check the value of the name at the top of the sheet.

3. **(2:05 P.M.)** 215 minutes is 3 hours and 35 minutes. Counting 3 hours and 35 minutes from 10:30 AM gives 2:05 PM.

4. **(20%)** If 4 out of 5 cars have a tape player or radio, then 1 out of 5 do not. One out of five is written 1/5, and another name for this fraction is 2/10 or 20/100, which is 20%.

5. **(⁻110, 25, and ⁻73.4 should be circled)** Students can use their number sense to judge where these real numbers should be on a number line. ⁻110 would be to the left of ⁻100. 25 is positive since there's no negative sign in front of it, so it goes to the right of 0. ⁻73.4 is negative, and hence would be to the left of zero.

6. **(125)** The volume of the large cube is 5 x 5 x 5 = 125 cubic inches. The volume of the small cube is 1 x 1 x 1 or 1 cubic inch. Therefore 125 small cubes fit into the large cube.

7. **(13)** Assume the worst case: you might draw 3 clubs, 3 spades, 3 hearts and 3 diamonds, a total of 12 cards, and not have four of one suit yet. On the 13th draw, you will draw the 4th heart, club, diamond or spade.

8. **(Rowena: 16; Jim: 12)** A student might *guess-check-revise* until they find x and y such that $x + y = 28$ and $x - y = 4$. Guess $x = 20$, and then y must be 8 for the first condition to hold. But 20 - 8 isn't 4, so $x = 20$ might be revised down. Continue in this fashion.

Commentary

Neptune, XVI

1. **(18%)** 27 + 24 = 47 honor students. There were 47 + 210 students in all. Therefore the percent of honor students is 47 out of 257, which is 47 ÷ 257 or approximately 18%.

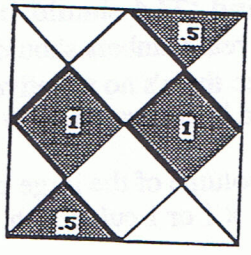
2. **(130)** $60 \div \frac{1}{2}$ is $60 \times \frac{2}{1}$ or 120. 120 + 10 = 130.

3. **(1350)** One approach is to convert 8 inches to $\frac{8}{12}$ or $\frac{2}{3}$ of a foot, which is 0.67 feet when written as a decimal. 6 feet 6 inches is 6.5 feet, when written as a decimal. The volume of the mattress is then 0.67 feet x 5 feet x 6.5 feet, which can be computed on a calculator easily, giving 21.775 cubic feet. This is multiplied by 62 pounds per cubic foot, given 1350 pounds as the weight of the water bed mattress. Students might now realize why many buildings won't allow water beds on the second floor of an apartment -- with the frame and two people in the bed, it's close to a ton.

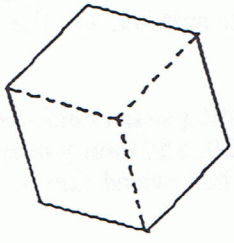
4. **(21)** $3\frac{1}{4}$ miles = $\frac{13}{4}$ miles. The depth drops one foot for every $\frac{1}{4}$ mile, so it drops 13 feet. 13 + 8 = 21 feet deep.

5. **(21)** It will take 4 cuts to cut a log into 5 pieces. $5\frac{1}{4}$ minutes x 4 gives $20\frac{4}{4}$ or 21 minutes.

6. **(37.5%)** The isosceles triangles are also half squares. Putting together half squares and squares to make whole squares, the shaded part is three squares. The whole area consists of 8 whole squares, and 3 out of 8 = $\frac{3}{8}$ = .375 = 37.5%



7. **(See the figure below.)**



8. **(5)** As bites are taken, the weight goes down by half each time. The amount remaining after each bite can then be found by multiplying what remains by 0.5, as shown in the chart below.

0.5 x 4 = 2	*Bite 1*	0.5 x 0.5 = 0.25	*Bite 4*
0.5 x 2 = 1	*Bite 2*	0.5 x 0.25 = 0.125	*Bite 5*
0.5 x 1 = 0.5	*Bite 3*		

Commentary

Neptune, XVII

1. **($3.00)** $8.00 - $4.25 = $3.75 for the extra 3 people. Therefore $1.25 is the charge for each additional person. Working with $4.25 for 3 people, and $1.25 is charged for person #3, we know that $3.00 is the charge for the driver and 1 passenger.

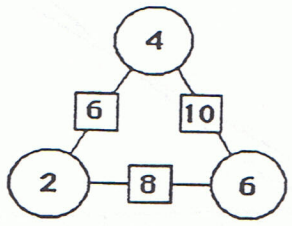
2. **(15)** Seven are always at the laundry; he picks up seven clean ones for Tuesday - Monday; and he is wearing a shirt when he drops off the laundry.

3. **(60)** From 6:00 A.M. until 9:00 P.M. is 15 hours. If they eat for 3 hours and spend 2 hours buying gas, the actual driving time is 15 - 5 = 10 hours. 600 miles ÷ 10 hours = 60 miles per hour.

4. **(24)** Students might make an organized list, using letters of the alphabet to represent the crust, the cheese, and the toppings. If so, they should get 3 x 2 x 4 = 24 varieties.

5. **(343)** 294 ÷ 6 means that the area of each square is 49 square units. If the area of each square is 49 square units, the side length must be 7 units. The volume of a box is length times width times height, or 7 x 7 x 7.

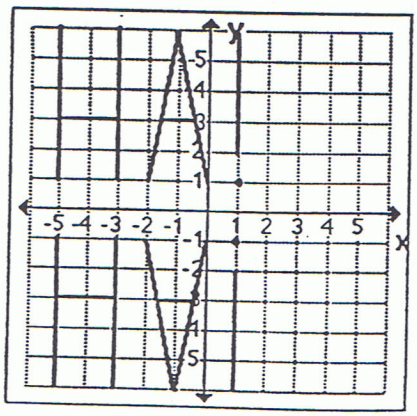
6.



7. **(One)** Use *trial and error*, or a calculator, to find 69545. The possible zip codes are 69141, 69242, 69343, 69444, 69545, 69646, 69747, 69848, 69949, and 69040. The only one divisible by 7 is 69545.

8. **(8,8,8,8; 8,8,9,7; 9,9,7,7; 9,9,9,5; 9,9,8,6)** If the average of 4 whole numbers is 8, the sum of the 4 numbers must equal 32. Given that each number is less than 10, we have 8 + 8 + 8 + 8 = 32, 8 + 8 + 9 + 7 = 32, 9 + 9 + 7 + 7 = 32, 9 + 9 + 9 + 5 = 32, 9 + 9 + 8 + 6 = 32 as the only possibilities.

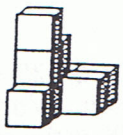
9. **(HA! is the word.)** You should see this:



Commentary

Neptune, XVIII

1. (9 cubes) 6 on bottom, 2 on second level, 1 on top.



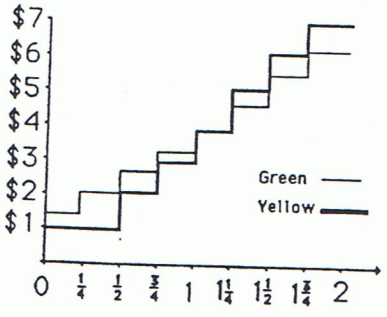
2. (30°) 360° ÷ 12 sections = 30°.

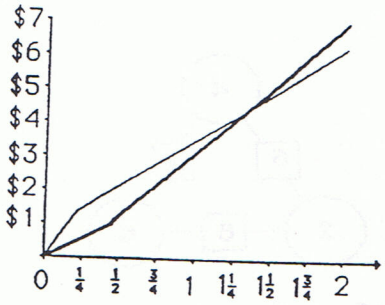
3. (2748) The perimeter of the building would be 5 times the length of a side, so the perimeter would be a multiple of 5. The only number given which is not a multiple of 5 is 2748.

4. (16/100) The chances are the number of tickets that both have together, divided by the total number. The answer can be expressed in a number of equivalent forms: 0.16, 16%, 8/50, 4/25, etc.

5. **(a. 1 1/4 miles; b. the lines coincide or cross)** Students should get a star for filling out the chart properly, for making a graph, and for answering each of the two questions. Most students will make a double line graph instead of a step graph, which is technically correct since the fares go up suddenly, rather than gradually, at each quarter mile. However, give students credit for either type of graph, if done correctly. Both types are shown below.

Cost of:	Green	Yellow
1/4 mile	$1.40	$1.00
1/2 mile	2.05	1.00
3/4 mile	2.70	2.00
1 mile	3.35	3.00
1 1/4 miles	4.00	4.00
1 1/2 miles	4.65	5.00
1 3/4 miles	5.30	6.00
2 miles	5.95	7.00





6. **(The new price is $1.20 less.)** Reducing $30 by 20% means the reduced price is $24. Increasing $24 by 20% can be done easily by multiplying 24 by 120% or 1.2 on a calculator, giving $28.80. $28.80 is $1.20 less than $30.

7. **(4)** There are several clues to consider in the teacher's rhyme. The first sentence says that there is one more fish than dish. The beginning of the second sentence implies there is an even number of fish; otherwise she couldn't place *two* fish upon each dish. So students might begin by guessing and checking from this list, finding that 4 fish and 3 dishes fit the bill.

fish	dish
2	1
4	3
6	5
.	.
.	.
.	.

Commentary

Neptune, XIX

1. (C) For each whole number from 0 to 19 substituted for x, the value of $\frac{X}{2}$ is less than 10.

2. **(a. 1; b. 7/16; c. 4; d. 28/16)** Since the area of the 1st square is 1/16, each side of the square must be 1/4 of an inch in length. So the perimeter of figure 1 is 4/4 or 1 inch. The 3rd figure is made from 7 squares, so its area is 7/16 inch^2. Its perimeter can be found by counting the exposed edges, resulting in 16 of the 1/4 inch lengths or 4 inches total. The figures beyond the ones seen can each be formed by adding 3 squares to the preceding figure. Counting out to the tenth such figure, there would be 28 squares, each 1/16 inch^2 in area. Therefore the 10th figure has area 28/16 inches^2, or 1 12/16 inches^2, or 1 3/4 inches^2.

3. **(4)** If 1 inch = 16 miles, then $\frac{1}{8}$ inch = $\frac{1}{8}$ of 16 miles or 2 miles. Therefore, the area equals 2 miles x 2 miles or 4 square miles.

4. **(2)** If $Y + 1 = 5$, then $Y = 4$. Since $X + 2 = Y$ also, then $X + 2 = 4$, or $X = 2$.

5. **(D $\frac{5N}{4}$)** Set up a proportion: $\frac{280}{N} = \frac{350}{X}$. Then the relationship between X and N can be determined. $280X = 350N$

$$X = \frac{350N}{280}$$

$$X = \frac{5N}{4}$$

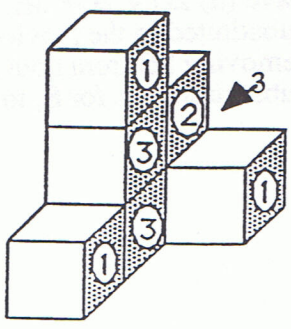
(Alternative: Simplify the fraction $\frac{350}{280} = \frac{5}{4}$.)

6. **(50%)** 30 + 6 gives 36 students in class, of which 12 + 6 or 18 are boys. Therefore, $\frac{18}{36} = 0.5 = 50\%$

7. **(C = 8) Students can try the various digits from 1 to 9 for B, and see if the digit gives a true statement. If so, students will find that 9 is the only such digit.**

$$\begin{array}{r} 99 \\ +99 \\ \hline 198 \end{array}$$

8. **(28) There are 6 faces on each block, and 7 blocks, resulting in 42 faces altogether. However, some of them are facing other faces and so will have no paint on them. Students might think of each cube individually, and count the number of faces it has that touch other faces, then get a total number of such faces, and subtract from 42. We counted 14 such touching faces, as shown on the block to the right.**



Commentary

Neptune, XX

1. **(10 1/2)** There are several approaches to finding the area. One is to find the area of the whole park first by filling in the grid lines and counting 16 acres. Then subtract the acres and half acres that aren't shaded -- 5 1/2 -- leaving 10 1/2 acres for the sports.

2. **(D)** There is 1 chance out of ten the number is 5 and 1 chance out of ten the number is 9. The chance the number is odd is 5 out of ten, and the chance the number is *not* 1 is 9 out of ten.

3. **(C)** The difference between 2 and 3 is 1, so (a) isn't the correct choice. The difference between 3 and 5 is 2, so (b) isn't correct. The difference between 5 and 13 is 8, so (d) isn't correct. (c) is the only other choice, and it is to be true. For the difference of two primes to be an odd number, one of the primes would have to be even but there's only one even prime -- 2 -- and the difference of two and any prime greater than 9 would have a difference greater than 7.

4. **(Median)** The mode (most often occurring number) is 88. The median (middle) number (after numbers are put in order) is between 88 and 90 or 89. The mean is the sum of the numbers -- 528 -- divided by 6, or 88. So the median is the best option for Lin.

5. **(18 and 40)** Starting with 7, the next term is 2 x 7 + 4, or 18. The following term is 2 x 18 + 4, or 40. An extension of the problem might be to ask what number could have come before 7.

6. **(Penny, Nickel, Quarter, Dime)** Students can *guess-check-revise* to find the order.

7. **(4096)** The sequence appears to be the powers of 8, in order: $8^1 = 8$; $8^2 = 64$; $8^3 = 512$ $8^4 = 4096$

8. **(32)** $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{8}{6} = \frac{32}{24}$. Note that some students will be tempted to say $\frac{8}{24}$, since it appears that the denominators of the fractions have been added to get the denominator 24; the temptation for students is to then add the numerators.

9. **(5)** There are many ways that students will approach this problem. Let the first initial stand for each animal – D for dinosaur, L for lion, E for elephant, and B for burro -- and what the scales show are these:

   (a) D + L = B (b) D = E + L (c) 2B = 3E

   Doubling (a), we know that 2D + 2L = 2B. But from (c), 2B = 3E, so combining these, we have (d) 2D + 2L = 3E. From doubling (b), we have that 2D = 2E + 2L. Now 2E + 2L can be substituted in the previous statement (d) to give 2E + 2L + 2L = 3E, or 2E + 4L = 3E. By removing 2E from both sides, we're left with 4L = E. This fact can then be used with (b), substituting 4L for E, to give D = 4L + L, or D = 5L.

Commentary

Neptune, XXI

1. **(27th floor)** $27 + 3 + {}^{-}5 + 7 + {}^{-}9 = 23$

2. **(Figure A)** Figure A has 36 squares as surfaces, and Figure B has 32 squares as surfaces.

Figure A	
Front	10
Back	10
Top	4
Bottom	4
Sides	8
Total:	36

Figure B	
Front	5
Back	5
Top	5
Bottom	5
Sides	12
Total:	32

3. **(42)** Students can approach this in a number of ways. *Guess-check-revise* for the starting number is one way; another is *working backward*. For the latter, if $x \div 6 - 3 = 4$, then $x \div 6$ must be 7 since $7 - 3 = 4$. But if $x \div 6 = 7$, then x must be 42 since $42 \div 6 = 7$.

4. **(D, 462)** The perimeter of a square is $4s$, where s is the length of one side. This mean the perimeter of a square must be divisible by 4. All of the numbers given except 462 are divisible by 4, with no remainder. But $462 \div 4 = 115.5$, so (d) is the only choice.

5. **(Acute)** An obtuse angle and its supplement equal 180°, and the obtuse angle is >90°. Therefore its supplement must be acute (<90°).

6. $(\frac{3}{5})$ Students can add the two numbers shown and divide by two to find the midpoint.

$$\frac{4}{5} + \frac{4}{10} = \frac{12}{10} \text{ and } \frac{1}{2} \times \frac{12}{10} = \frac{6}{10} \text{ or } \frac{3}{5}.$$

7. **(49)** Each 2-inch square tile is a square 2 inches on a side. Seven tiles fit across and 7 tiles fit down. The total number of tiles is then 7×7 or 49.

8. **(6)** John has 4 more brothers than sisters. One way to solve the problem in a concrete fashion would be to assign letters to John's brothers, perhaps A, B, C, D, and E, and let Mary be the only sister. Notice this matches the original condition that John has 4 more brothers (5) than sisters (1). Then the question is how many more brothers than sisters does Mary have? She has 6 brothers, counting John, and no sisters, so she has 6 more brothers than sisters.

9. **(a. 24; b. 27; c. 306; d. $3 \times (n + 2)$)** Students will likely note that you get from one figure to the next by adding 3 each time. It's therefore easy to get the total for figures 6 and 7. The total for figure 100 probably requires a generalization, looking for the connection between the figure number n and the total number of dots. (d) gives students a chance to express this generalization algebraically. Notice for alternatives to "$3 \times (n + 2)$," such as "$3 \times n + 6$" and "$n + n + n + 6$."

Commentary

Neptune, XXII

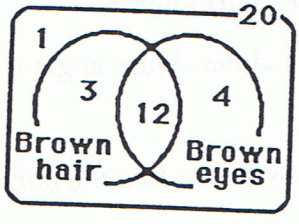
1. **(4.9)** All five scores, when added, must equal 5 x 4.6 or 23. We know 4.2 + 4.3 + 4.6 + 5.0 = 18.1, so the missing number is 23 - 18.1 or 4.9.

2. **(B)** $\frac{4}{9}$, $\frac{4}{8}$, $\frac{4}{7}$ Students should be encouraged to solve this using number sense. For example, in (a) thinking of the fractions as mixed numbers helps to see the answer pretty quickly. For (b) and (c), the numerators are all the same, so the fraction with the smallest denominator is the largest. For (d), the center number is 1/2, which is less than both of the other two. Some students might forget about number sense and find decimal equivalents first using a calculator (0.4444. . ., 0.5, 0.5714286 ..), and compare the numbers that way.

3. **(10.5 miles)** One-fourth of 2 hours is 30 minutes of walking; three-fourths of 2 hours is 90 minutes of running. The running distance, at 1 mile per 10 minutes, is 9 miles for 30 minutes. The walking distance, at 1 mile per 20 minutes, is 1.5 miles for 30 minutes.

4. **(200)** A strategy would be to "work backward". If 10% of a number is 2, that number must be 20. If 10% of x is 20, then x must be 200.

5. **(1 student)** One strategy is to construct a Venn diagram, working from the overlap area to the outside. I.e., first you insert 12 into the overlap area, and work toward the outside.



6. **(As shown to the right.)**

7. **(a. 23; b. $2n + 3$)** As students seek the answer to (a), they will probably simply continue to count the odd numbers till they reach the 10th odd number after 5. Part (b) forces the student to search for a way to express the function algebraically, using the variable n. Watch for alternative versions of $2n + 3$, such as $n + n + 3$ or $(n + 1) + (n + 2)$. If a student writes in English how to find the number for the nth figure, instead of as an algebraic expression, give the student 1 star instead of 2.

8. **(14)** 7 cubes have 6 x 7 or 42 faces, 28 of which are painted. 42 - 28 = 14 remain unpainted.

9. **(29)** 170 ÷ 6 = 28.3, so 29 rows are needed.

Commentary

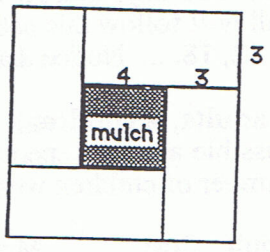
Neptune, XXIII

1. **(10:40 A.M.)** The hands would show the normal position for 10:40.

2. **(a. clockwise; b. counterclockwise; c. clockwise; d. clockwise)** Hopefully students will notice a pattern in this problem. An extension of the problem for the classroom is to define how the *n*th gear would turn, which depends on whether *n* is odd or even.

3. **(12)** Students might approach this by making an organized list. Using the first initial of each course, the list might start with {(F, L, v), (F,-L, s), ...}. As there are 2 choices for language, 2 for computer course, and 3 for physical education class, there will be 2 x 2 x 3 items in the list.

4. **(7, 3. The drawing is shown to the right.)** From the drawing, with the 4 m square of mulch sitting in the center, the distance from the mulch to the out edge of the garden is 3 m. From this, the width of the rectangle is 3 m. The length of each is then 3 m + 4 m, or 7 m.



5. **(30 miles per hour)** 20 miles in 40 minutes is 10 miles in 20 minutes. Taking this rate 3 times means you could travel 30 miles in 60 minutes, or, 30 miles per hour.

6. **(28)** Students might find this by guess-check-revise, continually guess what *x* might be until they are successful with a check. However, they might decide to simplify the equation to $X = 14 + \frac{1}{2}X$, and solve by subtracting $\frac{1}{2}X$ from both sides, getting $\frac{1}{2}X = 14$, or $X = 28$.

7. **(B)** The pattern is determined by counting by tens in the negative direction. Therefore once you start with -10, -20, -30, ..., you can simply switch to the positive numbers and count by tens as 10, 20, 30, ... and decide where 500 would be in such a count. If you count several such rows, you'll notice that column F is the multiples of 6, with a negative sign and zero added (-60, -120, -180, ...). The closest such number to -500 would be -480, so -480 would go in column F. This means -490 is in A, and -500 is in B.

8. **($3b + 31 = 47.2$; 5.4 grams)** Students should see that there are 3 balls on the scale, so $b + b + b$ gives the weight of the 3 balls alone. This amount, added to 31, gives 47.2. This equation might be written as shown above, or in a number of alternate ways, such as $3b + 31 = 47.2$. A student might solve this equation, or they might solve the problem intuitively by removing 31 from the scale and calculating what the new scale would show, then dividing this amount into 3 equal piles. In either case, the answer would be 5.4.

Commentary

Neptune, XXIV

1. (2003) On a calculator, students can use the repeating function concept to repeatedly multiply by 90%, finding the value for each new year in one step. Seven years from 1996 (2003) the car will have a value less than $21,000.

2. **(17)** Denise might put one apple in the first bag, two apples in the second bag, three apples in the third bag, and so on. By the sixteenth bag, 136 apples will have been used. If you put 17 in the next bag, which the pattern leads you to do, you would then have 17 left for the 18th bag but this would repeat the number seventeen. However, by placing 34 apples in the 17th bag, the problem is solved.

3. **(450)** The hare's rate of 9 meters per minute is 9 x 60 or 540 meters per hour. Likewise, the tortoise's rate is 1.5 x 60 or 90 meters per hour. So, in one hour the hare was 540 - 90 or 450 meters ahead. Some students may figure that the hare's rate is 7.5 m/hr faster, so 7.5 X 60 = 450 meters.

4. **(12)** The days that Al will call will follow this pattern: 3, 6, 9, 12, ... The days that Bob will call will follow this pattern: 4, 8, 12, ... The days that Chris will call will follow this pattern: 6, 12, 18, ... Notice that 12 is common to all 3 patterns, so that is a day that all three will call.

5. **(3 adults, 4 children)** A possible approach to this problem is to make a chart. Fill in possible amounts spent on adult tickets. Decide for each if it's possible to have a whole number of children with the remainder. The only adult possibility is 3 with 4 children.

Adult	Cost	Money left	$ left for Children ÷ ticket cost
1	$12.00	$56 - $12 = $44	$44 ÷ $5 = 8.8 children (not possible)
2	$24.00	$56 - $24 = $32	$32 ÷ $5 = 6.4 children (not possible)
3	$36.00	$56 - $36 = $20	$20 ÷ $5 = 4 children (possible)
4	$48.00	$56 - $48 = $8	$8 ÷ $5 = 1.6 children (not possible)

6. **(One possible solution: (4 + 4 + 4) ÷ 4 = 3)** Answers may vary; students can try various operations and record the results.

7. **(59)** The lowest average would come from making 0 on the last test. The average would then be (92 + 84 + 0) ÷ 3 = 58.666. . . or 59 to the nearest whole number.

8. **($\frac{8}{52}$ or $\frac{2}{13}$ or 0.15 or 15%)** There are 4 fours and 4 eights, a total of 8 number cards out of 52 that are evenly divisible by 4. Therefore the chances are 8/52 or one of its equivalent forms.

Commentary

Neptune, XXV

1. **(90)** The highest possible average would mean you made 100% on the last test. Then you average would be (83% + 91% + 86% + 100%) ÷ 4, or 90%.

2. **(One answer: (4 x 4 + 4) ÷ 4)** Students will likely try various combinations and compute each to see if it is correct.

3. **(Letters above the line are: T, V, W, X, Y, and Z. The others, P, Q, R, S, and U are below the line.)** Some students will get this immediately, and others will struggle trying to find a numerical pattern that underlies the placement of the letters. The rule used is that letters made strictly with line segments are above the line – those with curves go below the line. If a student does find a numerically-based pattern so that all of the given examples fit, they should get credit for the problem.

4. **($4\pi^2$ or about 39)** The circumference of the circle is given by πd, where d is the diameter. In this case, the circumference is 8π. If this is also the perimeter of the square, then each side is $8\pi \div 4$ or 2π. Then the area of the square is $(2\pi)^2$ or $4\pi^2$.

5. **(97)** One pattern is that you add 3, then 6, then 12, then 24, doubling what you add each time to get the next number. Another way to describe the pattern is that you double each number to get the next term, and subtract 1. Students may notice other ways to describe this pattern.

6. **(8, 8, and 8)** If you choose one radius shown, you can count one acute angle for the next radius you meet proceeding. Then you move to the next radius and proceed around the circle clockwise until you return to the starting radius. The same procedure will show that there are the same number of right and obtuse angles.

7. **(3,000,000,000)** For a female: 70 x 60 x 24 x 365 x 79 = 2,906,600,000 -- for a male: 70 x 60 x 24 x 365 x 75 = 2,759,400,000. Each of these numbers rounds to 3 billion.

8. **(Sam)** One way to begin is to ask "What happens if someone is lying?" For example, if Sue is lying, then all three are lying by looking at the other two clues. But all three lying can't happen because of the conditions. So Sue is telling the truth and didn't take the cookies. Then if Sam is lying, he must have taken the cookies because we know that Sue didn't – this means Bob is lying also, which is possible. Could Sam be telling the truth? If so, then Bob is also telling the truth, which is impossible because all three can't be telling the truth by the conditions. The only possibility, then, is that Sam took the cookies and lied, Bob lied also, and Sue told the truth.

Commentary

Neptune, XXVI

1. (Possible answer: $(4 + 4) \div 4 + 4 = 6$)

2. **(About 51%)** The volume of the larger cube is 5^3 or 125 cubic inches. The volume of the smaller cube is 4^3 or 64 cubic inches. $64 \div 125 = 0.512$ or about 51%.

3. **(about 67%)** They won 12 games out of 18 played. $12 \div 18$ is 0.666..., which rounds off to 67%, to the nearest whole percent.

4. (8) Six laps in 12 minutes means 1 lap in 2 minutes. Therefore 4 laps would take 8 minutes.

5. **(1/16)** 1 quart = 2 pints = 4 cups = 1/4 gallon: Four quarts is 1 gallon, so one quart is 1/4 of a gallon. Each ate 1 cup. So 1/4 of 1/4 of a gallon would be 1/16 of a gallon.

6. **(Angles 2, 3, 6, & 7 = 60°; angles 1, 4, 5, & 8 = 120°.)** These angles show what happens when a transversal line cuts a pair of parallel lines.

7. ($\frac{1}{9}$) The first day, there is a 1/3 chance of getting the grand prize. So this should happen 1/3 of the time. Given that it does happen, there's a 1/3 chance that you'll open the right door the second time also. This results in 1/3 of 1/3 of the time that both will hold true, or 1/9 of the time.

8. **(1.69)** (A calculator will help students get this answer. 2.5 mm/week x 52 weeks/year x 13 years is 1690 mm. This is 1.69 meters.

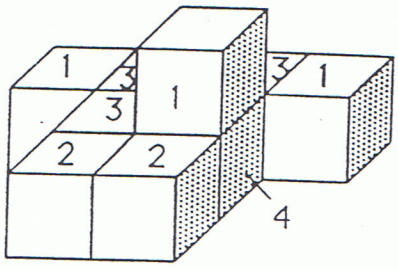
9. **(alligator)** If the circumference of the belt initially is 25,000 miles, it is 132,000,000 feet. The radius from the center of the earth would then be found by using $C = 2\pi r$. This radius is 21,019,108 feet. With 10 feet added to the circumference, producing 132,000,010 feet, the radius of the loosened belt would be 21,019,109.5 or about a foot and a half more than before. This would be room for an alligator to crawl under.

Commentary

Neptune, XXVII

1. **(Sample answer: 4 + 4 - (4 ÷ 4) = 7)**

2. **(20)** One way to approach this problem is to consider each cube in turn, asking how many faces it has that are glued to another face. The numbers on the cube faces show the numbers for each cube. Their sum is 20.



3. **(3:54)** One inch per minute is a rate of 1 foot in 12 minutes. The depth must go from 3 1/2 feet to 8 feet, which is 4 1/2 feet. Therefore the number of minutes required is (4 1/2) x 12 or 54 minutes.

4. **(91)** Students might begin by looking at the multiples of 7, and checking to see which of those have a remainder of 1 when divided by 3, 5, and 6. The first of these is 91. (91 ÷ 3 = 30 R 1; 91 ÷ 5 = 18 R 1; 91 ÷ 6 = 15 R 1; 91 ÷ 7 = 13, no remainder)

5. **(34, 55, 89, 144)** The pattern is the widely-known Fibonnaci sequence, which starts with 1, 1 and from there on, each term is the sum of the two preceding terms. This sequence has been linked to many natural occurrences on earth.

6. **(490)** The largest possible such integer would be paired with the four smallest other positive integers. Therefore this number is paired with 1, 2, 3, and 4, and we know it must be 490 since 490 + 1 + 2 + 3 + 4 = 500.

7. **(50)** 200 ÷ 4 = 50 letters per typist for 2 days, or 25 letters per day per typist. So 2 typists can do 50 letters per day.

8. **(6)** Denote the three members by A, B, and C. The ways they can be arranged are then: ABC, ACB, BCA, BAC, CAB, CBA.

9. **(Equation: $37 = 1/2 + 1.5y$; $y = 24.3$)** Students might write alternate versions of the equation, and 24.3 has been rounded to the nearest decimal. The equation can be solved by intuition or *guess-check-revise*, or by a formal procedure that involves subtracting 1/2 from both sides, then dividing both sides by 1.5, to isolate the variable x.

10. **(Equation: $88 = 10 + 52\frac{a}{2}$; $a = 3$)** Again, students might write alternate forms of the equation. The equation can be solved by intuition or *guess-check-revise*, or by a formal procedure that involves subtracting 10 from both sides, then dividing both sides by 26, to isolate the variable a.