Section 3: Math Test – No Calculator

QUESTION 1

Choice B is correct. Subtracting z from both sides of $2z + 1 = z$ results in $z + 1 = 0$. Subtracting 1 from both sides of $z + 1 = 0$ results in $z = -1$.

Choices A, C, and D are incorrect. When each of these values is substituted for z in the given equation, the result is a false statement. Substituting -2 for z yields $2(-2) + 1 = -2$, or $-3 = -2$. Substituting $\frac{1}{2}$ for z yields $2\left(\frac{1}{2}\right) + 1 = \frac{1}{2}$, or $2 = \frac{1}{2}$. Lastly, substituting 1 for z yields $2(1) + 1 = 1$, or $3 = 1$.

QUESTION 2

Choice C is correct. To complete the purchase, the initial payment of \$60 plus the w weekly payments of \$30 must be equivalent to the \$300 price of the television. The total, in dollars, of w weekly payments of \$30 can be expressed by $30w$. It follows that $300 = 30w + 60$ can be used to find the number of weekly payments, w, required to complete the purchase.

Choice A is incorrect. Since the television is to be purchased with an initial payment and w weekly payments, the price of the television must be equivalent to the sum, not the difference, of these payments. Choice B is incorrect. This equation represents a situation where the television is purchased using only w weekly payments of \$30, with no initial payment of \$60. Choice D is incorrect. This equation represents a situation where the w weekly payments are \$60 each, not \$30 each, and the initial payment is \$30, not \$60. Also, since the television is to be purchased with weekly payments and an initial payment, the price of the television must be equivalent to the sum, not the difference, of these payments.

QUESTION 3

Choice B is correct. Since the relationship between the merchandise weight x and the shipping charge $f(x)$ is linear, a function in the form $f(x) = mx + b$, where m and b are constants, can be used. In this situation, the constant m represents the additional shipping charge, in dollars, for each additional pound of merchandise shipped, and the constant b represents a one-time charge, in dollars, for shipping any weight, in pounds, of merchandise. Using any two pairs of values from the table, such as (10, 21.89) and (20, 31.79), and dividing the difference in the charges by the difference in the weights gives the value of m: $m = \frac{31.79 - 21.89}{20 - 10}$, which simplifies to $\frac{9.9}{10}$, or 0.99. Substituting the value of m and any pair of values from the table, such as (10, 21.89), for x and $f(x)$, respectively, gives the value of b: $21.89 = 0.99(10) + b$, or $b = 11.99$. Therefore, the function $f(x) = 0.99x + 11.99$ can be used to determine the total shipping charge $f(x)$, in dollars, for an order with a merchandise weight of x pounds.

Choices A, C, and D are incorrect. If any pair of values from the table is substituted for x and $f(x)$, respectively, in these functions, the result is false. For example, substituting 10 for x and 21.89 for $f(x)$ in $f(x) = 0.99x$ yields $21.89 = 0.99(10)$, or $21.89 = 9.9$, which is false. Similarly, substituting the values (10, 21.89) for x and $f(x)$ into the functions in choices C and D results in $21.89 = 33.9$ and $21.89 = 50.84$, respectively. Both are false.

QUESTION 4

Choice C is correct. It's given that the graph represents $y = h(x)$, thus the y-coordinate of each point on the graph corresponds to the height, in feet, of a Doric column with a base diameter of x feet. A Doric column with a base diameter of 5 feet is represented by the point (5, 35), and a Doric column with a base diameter of 2 feet is represented by the point (2, 14). Therefore, the column with a base diameter of 5 feet has a height of 35 feet, and the column with a base diameter of 2 feet has a height of 14 feet. It follows that the difference in heights of these two columns is $35 - 14$, or 21 feet.

Choice A is incorrect. This value is the slope of the line and represents the increase in the height of a Doric column for each increase in the base diameter by 1 foot. Choice B is incorrect. This value represents the height of a Doric column with a base diameter of 2 feet, or the difference in height between a Doric column with base diameter of 5 feet and a Doric column with base diameter of 3 feet. Choice D is incorrect and may result from conceptual or calculation errors.

QUESTION 5

Choice A is correct. The expression $\sqrt{9x^2}$ can be rewritten as $(\sqrt{9})(\sqrt{x^2})$. The square root symbol in an expression represents the principal square root, or the positive square root, thus $\sqrt{9} = 3$. Since $x > 0$, taking the square root of the second factor, $\sqrt{x^2}$, gives x. It follows that $\sqrt{9x^2}$ is equivalent to $3x$.

Choice B is incorrect and may result from rewriting $\sqrt{9x^2}$ as $(\sqrt{9})(x^2)$ rather than $(\sqrt{9})(\sqrt{x^2})$. Choices C and D are incorrect and may result from misunderstanding the operation indicated by a radical symbol. In both of these choices, instead of finding the square root of the coefficient 9, the coefficient has been multiplied by 2. Additionally, in choice D, x^2 has been squared to give x^4, instead of taking the square root of x^2 to get x.

QUESTION 6

Choice A is correct. Factoring the numerator of the rational expression $\frac{x^2 - 1}{x - 1}$ yields $\frac{(x + 1)(x - 1)}{x - 1}$. The expression $\frac{(x + 1)(x - 1)}{x - 1}$ can be rewritten as $\left(\frac{x + 1}{1}\right)\left(\frac{x - 1}{x - 1}\right)$. Since $\frac{x - 1}{x - 1} = 1$, when x is not equal to 1, it follows that $\left(\frac{x + 1}{1}\right)\left(\frac{x - 1}{x - 1}\right) = \left(\frac{x + 1}{1}\right)(1)$ or $x + 1$. Therefore, the given equation is equivalent to $x + 1 = -2$. Subtracting 1 from both sides of $x + 1 = -2$ yields $x = -3$.

Choices B, C, and D are incorrect. Substituting 0, 1, or −1, respectively, for x in the given equation yields a false statement. Substituting 0 for x in the given equation yields $\frac{0^2 - 1}{0 - 1} = -2$ or $1 = -2$, which is false. Substituting 1 for x in the given equation makes the left-hand side $\frac{1^2 - 1}{1 - 1} = \frac{0}{0}$, which is undefined and not equal to −2. Substituting −1 for x in the given equation yields $\frac{(-1)^2 - 1}{-1 - 1} = -2$, or $0 = -2$, which is false. Therefore, these values don't satisfy the given equation.

QUESTION 7

Choice D is correct. Since $y = f(x)$, the value of $f(0)$ is equal to the value of $f(x)$, or y, when $x = 0$. The graph indicates that when $x = 0$, $y = 4$. It follows that the value of $f(0) = 4$.

Choice A is incorrect. If the value of $f(0)$ were 0, then when $x = 0$, the value of $f(x)$, or y, would be 0 and the graph would pass through the point (0, 0). Choice B is incorrect. If the value of $f(0)$ were 2, then when $x = 0$, the value of $f(x)$, or y, would be 2 and the graph would pass through the point (0, 2). Choice C is incorrect. If the value of $f(0)$ were 3, then when $x = 0$, the value of $f(x)$, or y, would be 3 and the graph would pass through the point (0, 3).

QUESTION 8

Choice C is correct. Since point B lies on $\overline{AD}$, angles ABC and CBD are supplementary. It's given that angle ABC is a right angle; therefore, its measure is 90°. It follows that the measure of angle CBD is 180° − 90°, or 90°. By the angle addition postulate, the measure of angle CBD is equivalent to $x° + 2x° + 2x°$. Therefore, $90 = x + 2x + 2x$. Combining like terms gives $90 = 5x$. Dividing both sides of this equation by 5 yields $x = 18$. Therefore, the value of $3x$ is 3(18), or 54.

Choice A is incorrect. This is the value of x. Choice B is incorrect. This is the value of $2x$. Choice D is incorrect. This is the value of $4x$.

QUESTION 9

Choice C is correct. The equation defining any line can be written in the form $y = mx + b$, where m is the slope of the line and b is the y-coordinate of the y-intercept. Line ℓ passes through the point (0, −4), which is the y-intercept. Therefore, for line ℓ, $b = -4$. The slope of a line is the ratio of the difference between the y-coordinates of any two points to the difference between the x-coordinates of the same points. Calculating the slope using two points that line ℓ passes through, (−4, 0) and (0, −4), gives $m = \frac{0 - (-4)}{(-4) - 0} = \frac{4}{-4}$, or −1. Since $m = -1$ and $b = -4$, the equation of line ℓ can be written as $y = (-1)x + (-4)$, or $y = -x - 4$. Adding x to both sides of $y = -x - 4$ yields $x + y = -4$.

Choices A and B are incorrect. These equations both represent lines with a positive slope, but line ℓ has a negative slope. Choice D is incorrect. This equation represents a line that passes through the points (4, 0) and (0, 4), not the points (−4, 0) and (0, −4).

QUESTION 10

Choice D is correct. Since the graph represents the equation $y = 2x^2 + 10x + 12$, it follows that each point (x, y) on the graph is a solution to this equation. Since the graph crosses the y-axis at $(0, k)$, then substituting 0 for x and k for y in $y = 2x^2 + 10x + 12$ creates a true statement: $k = 2(0)^2 + 10(0) + 12$, or $k = 12$.

Choice A is incorrect. If $k = 2$ when $x = 0$, it follows that $2 = 2(0)^2 + 10(0) + 12$, or $2 = 12$, which is false. Choice B is incorrect. If $k = 6$ when $x = 0$, it follows that $6 = 2(0)^2 + 10(0) + 12$, or $6 = 12$, which is false. Choice C is incorrect. If $k = 10$ when $x = 0$, it follows that $10 = 2(0)^2 + 10(0) + 12$, or $10 = 12$, which is false.

QUESTION 11

Choice A is correct. A circle in the xy-plane with center (h, k) and radius r is defined by the equation $(x - h)^2 + (y - k)^2 = r^2$. Therefore, an equation of a circle with center (5, 7) and radius 2 is $(x - 5)^2 + (y - 7)^2 = 2^2$, or $(x - 5)^2 + (y - 7)^2 = 4$.

Choice B is incorrect. This equation defines a circle with center $(-5, -7)$ and radius 2. Choice C is incorrect. This equation defines a circle with center (5, 7) and radius $\sqrt{2}$. Choice D is incorrect. This equation defines a circle with center $(-5, -7)$ and radius $\sqrt{2}$.

QUESTION 12

Choice B is correct. Since figures are drawn to scale unless otherwise stated and triangle ABC is similar to triangle DEF, it follows that the measure of angle B is equal to the measure of angle E. Furthermore, it follows that side AB corresponds to side DE and that side BC corresponds to side EF. For similar triangles, corresponding sides are proportional, so $\frac{AB}{BC} = \frac{DE}{EF}$. In right triangle DEF, the cosine of angle E, or $\cos(E)$, is equal to the length of the side adjacent to angle E divided by the length of the hypotenuse in triangle DEF. Therefore, $\cos(E) = \frac{DE}{EF}$, which is equivalent to $\frac{AB}{BC}$. Therefore, $\cos(E) = \frac{12}{13}$.

Choice A is incorrect. This value represents the tangent of angle F, or $\tan(F)$, which is defined as the length of the side opposite angle F divided by the length of the side adjacent to angle F. Choice C is incorrect. This value represents the tangent of angle E, or $\tan(E)$, which is defined as the length of the side opposite angle E divided by the length of the side adjacent to angle E. Choice D is incorrect. This value represents the sine of angle E, or $\sin(E)$, which is defined as the length of the side opposite angle E divided by the length of the hypotenuse.

QUESTION 13

Choice C is correct. The x-intercepts of the graph of $f(x) = x^2 + 5x + 4$ are the points $(x, f(x))$ on the graph where $f(x) = 0$. Substituting 0 for $f(x)$ in the function equation yields $0 = x^2 + 5x + 4$. Factoring the right-hand side of $0 = x^2 + 5x + 4$ yields $0 = (x + 4)(x + 1)$.

If $0 = (x + 4)(x + 1)$, then $0 = x + 4$ or $0 = x + 1$. Solving both of these equations for x yields $x = -4$ and $x = -1$. Therefore, the x-intercepts of the graph of $f(x) = x^2 + 5x + 4$ are $(-4, 0)$ and $(-1, 0)$. Since both points lie on the x-axis, the distance between $(-4, 0)$ and $(-1, 0)$ is equivalent to the number of unit spaces between -4 and -1 on the x-axis, which is 3.

Choice A is incorrect. This is the distance from the origin to the x-intercept $(-1, 0)$. Choice B is incorrect and may result from incorrectly calculating the x-intercepts. Choice D is incorrect. This is the distance from the origin to the x-intercept $(-4, 0)$.

QUESTION 14

Choice B is correct. Squaring both sides of the equation $\sqrt{4x} = x - 3$ yields $4x = (x - 3)^2$, or $4x = (x - 3)(x - 3)$. Applying the distributive property on the right-hand side of the equation $4x = (x - 3)(x - 3)$ yields $4x = x^2 - 3x - 3x + 9$. Subtracting $4x$ from both sides of $4x = x^2 - 3x - 3x + 9$ yields $0 = x^2 - 3x - 3x - 4x + 9$, which can be rewritten as $0 = x^2 - 10x + 9$. Factoring the right-hand side of $0 = x^2 - 10x + 9$ gives $0 = (x - 1)(x - 9)$. By the zero product property, if $0 = (x - 1)(x - 9)$, then $0 = x - 1$ or $0 = x - 9$. Adding 1 to both sides of $0 = x - 1$ gives $x = 1$. Adding 9 to both sides of $0 = x - 9$ gives $x = 9$. Substituting these values for x into the given equation will determine whether they satisfy the equation. Substituting 1 for x in the given equation yields $\sqrt{4(1)} = 1 - 3$, or $\sqrt{4} = -2$, which is false. Therefore, $x = 1$ doesn't satisfy the given equation. Substituting 9 for x in the given equation yields $\sqrt{4(9)} = 9 - 3$ or $\sqrt{36} = 6$, which is true. Therefore, $x = 9$ satisfies the given equation.

Choices A and C are incorrect because $x = 1$ doesn't satisfy the given equation: $\sqrt{4x}$ represents the principal square root of $4x$, which can't be negative. Choice D is incorrect because $x = 9$ does satisfy the given equation.

QUESTION 15

Choice A is correct. A system of two linear equations has no solution if the graphs of the lines represented by the equations are parallel and are not equivalent. Parallel lines have equal slopes but different y-intercepts. The slopes and y-intercepts for the two given equations can be found by solving each equation for y in terms of x, thus putting the equations in slope-intercept form. This yields $y = 3x + 6$ and $y = \left(-\frac{a}{2}\right)x + 2$. The slope and y-intercept of the line with equation $-3x + y = 6$ are 3 and $(0, 6)$, respectively. The slope and y-intercept of the line with equation $ax + 2y = 4$ are represented by the expression $-\frac{a}{2}$ and the point $(0, 2)$, respectively. The value of a can be found by setting the two slopes equal to each other, which gives $-\frac{a}{2} = 3$. Multiplying both sides of this equation by -2 gives $a = -6$. When $a = -6$, the lines are parallel and have different y-intercepts.

Choices B, C, and D are incorrect because these values of a would result in two lines that are not parallel, and therefore the resulting system of equations would have a solution.

QUESTION 16

The correct answer is 2200. If the total shipping cost was \$47,000, then $T = 47{,}000$. If 3000 units were shipped to the farther location, then $f = 3000$. Substituting 47,000 for T and 3000 for f in the equation $T = 5c + 12f$ yields $47{,}000 = 5c + 12(3000)$. Multiplying 12 by 3000 yields $47{,}000 = 5c + 36{,}000$. Subtracting 36,000 from both sides of the equation yields $11{,}000 = 5c$. Dividing both sides by 5 yields $c = 2200$. Therefore, 2200 units were shipped to the closer location.

QUESTION 17

The correct answer is 5. By definition of absolute value, if $|2x + 1| = 5$, then $2x + 1 = 5$ or $-(2x + 1) = 5$, which can be rewritten as $2x + 1 = -5$. Subtracting 1 from both sides of $2x + 1 = 5$ and $2x + 1 = -5$ yields $2x = 4$ and $2x = -6$, respectively. Dividing both sides of $2x = 4$ and $2x = -6$ by 2 yields $x = 2$ and $x = -3$, respectively. If a and b are the solutions to the given equation, then $a = 2$ and $b = -3$. It follows then that $|a - b| = |2 - (-3)| = |5|$, which is 5. Similarly, if $a = -3$ and $b = 2$, it follows that $|a - b| = |-3 - 2| = |-5|$, which is also 5.

QUESTION 18

The correct answer is 1.21. It's given that each year, the value of the antique is estimated to increase by 10% over its value the previous year. Increasing a quantity by 10% is equivalent to the quantity increasing to 110% of its original value or multiplying the original quantity by 1.1. Therefore, 1 year after the purchase, the estimated value of the antique is 200(1.1) dollars. Then, 2 years after purchase, the estimated value of the antique is 200(1.1)(1.1), or 200(1.21) dollars. It's given that the estimated value of the antique after 2 years is $200a$ dollars. Therefore, $200(1.21) = 200a$. It follows that $a = 1.21$.

QUESTION 19

The correct answer is 2500. Adding the given equations yields $(2x + 3y) + (3x + 2y) = (1200 + 1300)$. Combining like terms yields $5x + 5y = 2500$. Therefore, the value of $5x + 5y$ is 2500.

QUESTION 20

The correct answer is 20. Factoring the expression $u^2 - t^2$ yields $(u - t)(u + t)$. Therefore, the expression $(u - t)(u^2 - t^2)$ can be rewritten as $(u - t)(u - t)(u + t)$. Substituting 5 for $u + t$ and 2 for $u - t$ in this expression yields (2)(2)(5), which is equal to 20.