CollegeBoard
Advanced Placement Program

AP® Calculus

Multiple-Choice Question Collection 1969–1998

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For more information about equity and access in principle and practice, please send an email to apequity@collegeboard.org.

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About This Collection

Multiple-choice questions from past AP Calculus Exams provide a rich resource for teaching topics in the course and reviewing for the exam each year. Over the years, some topics have been added or removed, but almost all of the old questions still offer interesting opportunities to investigate concepts and assess student understanding. Always consult the most recent Course Description on AP Central® for the current topic outlines for Calculus AB and Calculus BC.

Please note the following:

- The solution to each multiple-choice question suggests one possible way to solve that question. There are often alternative approaches that produce the same choice of answer, and for some questions such multiple approaches are provided. Teachers are also encouraged to investigate how the incorrect options for each question could be obtained to help students understand (and avoid) common types of mistakes.
- Scientific (nongraphing) calculators were required on the AP Calculus Exams in 1993.
- Graphing calculators have been required on the AP Calculus Exams since 1995. In 1997 and 1998, Section I, Part A did not allow the use of a calculator; Section I, Part B required the use of a graphing calculator.
- Materials included in this resource may not reflect the current AP Course Description and exam in this subject, and teachers are advised to take this into account as they use these materials to support their instruction of students. For up-to-date information about this AP course and exam, please download the official AP Course Description from the AP Central Web site at apcentral.collegeboard.com.

1969 AP Calculus AB: Section I

90 Minutes—No Calculator

Note: In this examination, ln *x* denotes the natural logarithm of *x* (that is, logarithm to the base *e*).

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

(A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$

(D) $f(x) = \log x$ (E) $f(x) = e^x$

---

2. $\ln(x-2) < 0$ if and only if

(A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$

(D) $x > 2$ (E) $x > 3$

---

3. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{3}$ (D) 1 (E) $\dfrac{7}{5}$

---

4. $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) 4 (E) 6

---

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\dfrac{dy}{dx}$ at $x = 1$ is

(A) –2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h \to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

---

7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

---

8. If $p(x) = (x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x - 1$, then $k =$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

---

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

---

10. The set of all points (e^t, t), where t is a real number, is the graph of $y =$

(A) $\frac{1}{e^x}$ (B) $e^{\frac{1}{x}}$ (C) $x e^{\frac{1}{x}}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

---

11. The point <u>on the curve</u> $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

12. If $f(x)=\dfrac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f\left(g(x)\right)=g\left(f(x)\right)$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

---

13. The region bounded by the x-axis and the part of the graph of $y=\cos x$ between $x=-\dfrac{\pi}{2}$ and $x=\dfrac{\pi}{2}$ is separated into two regions by the line $x=k$. If the area of the region for $-\dfrac{\pi}{2}\le x\le k$ is three times the area of the region for $k\le x\le\dfrac{\pi}{2}$, then $k=$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

---

14. If the function f is defined by $f(x)=x^5-1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x)=$

(A) $\dfrac{1}{\sqrt[5]{x}+1}$ (B) $\dfrac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

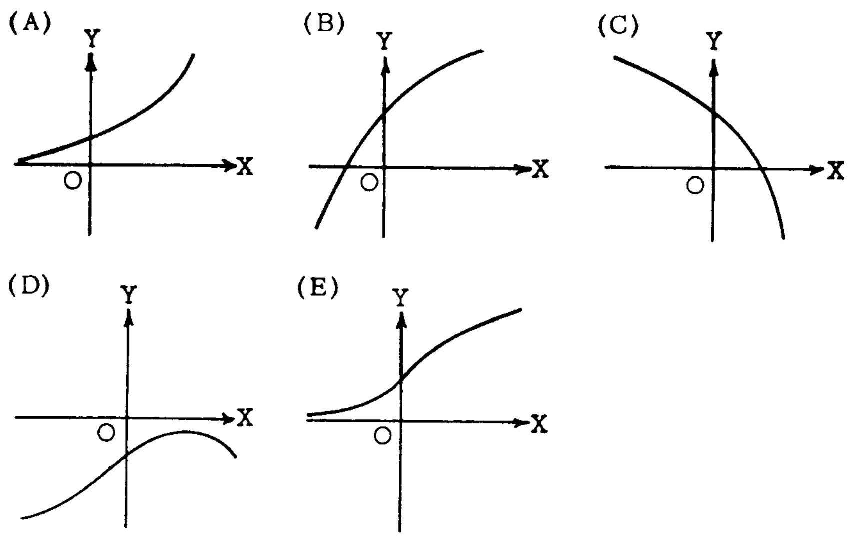
(D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

---

15. If $f'(x)$ and $g'(x)$ exist and $f'(x)>g'(x)$ for all real x, then the graph of $y=f(x)$ and the graph of $y=g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only
(B) $(3,162)$ only
(C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$
(E) $(0,0)$ and $(4,256)$

---

18. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1
(B) 0
(C) 1
(D) 2
(E) nonexistent

---

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \dfrac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1
(B) $e^{\frac{1}{2}}$
(C) e
(D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v.

20. An equation for a tangent to the graph of $y = \arcsin\frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

---

22. $\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

---

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

---

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

---

26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1
(E) none of the above

---

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$
(B) $\sec^2 x + C$
(C) $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$
(E) $\sec x \tan x + C$

---

28. The function defined by $f(x) = \sqrt{3}\cos x + 3\sin x$ has an amplitude of

(A) $3 - \sqrt{3}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) $3 + \sqrt{3}$ (E) $3\sqrt{3}$

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln \sqrt{2}$ (B) $\ln \frac{\pi}{4}$ (C) $\ln \sqrt{3}$ (D) $\ln \frac{\sqrt{3}}{2}$ (E) $\ln e$

---

30. If a function f is continuous for all x and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$.
(B) $f'(-1)=0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x=3$.
(E) The graph of f intersects both axes.

---

31. If $f'(x)=-f(x)$ and $f(1)=1$, then $f(x)=$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

---

32. If $a,b,c,d,$ and e are real numbers and $a \neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root.
(B) at least one real root.
(C) an odd number of nonreal roots.
(D) no real roots.
(E) no positive real roots.

---

33. What is the average (mean) value of $3t^3-t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

---

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

---

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

---

37. Which is the best of the following polynomial approximations to $\cos 2x$ near $x = 0$?

(A) $1 + \frac{x}{2}$ (B) $1 + x$ (C) $1 - \frac{x^2}{2}$ (D) $1 - 2x^2$ (E) $1 - 2x + x^2$

---

38. $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

---

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only
(D) nonzero n, only (E) all n

---

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

---

42. What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x-intercepts?

(A) All $k > 0$ (B) All $k < 4$ (C) $k = 0, 4$ (D) $0 < k < 4$ (E) All k

---

43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

---

44. The fundamental period of the function defined by $f(x) = 3 - 2\cos^2 \frac{\pi x}{3}$ is

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

---

45. If $\frac{d}{dx}\left(f(x)\right) = g(x)$ and $\frac{d}{dx}\left(g(x)\right) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\,g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. The asymptotes of the graph of the parametric equations $x = \frac{1}{t}$, $y = \frac{t}{t+1}$ are

(A) $x = 0,\ y = 0$ (B) $x = 0$ only (C) $x = -1,\ y = 0$
(D) $x = -1$ only (E) $x = 0,\ y = 1$

---

2. What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

(A) $(-1, 0)$ (B) $(0, 0)$ (C) $(0, 1)$ (D) $\left(1, \frac{\pi}{4}\right)$ (E) $\left(1, \frac{\pi}{2}\right)$

---

3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0, 0)$ and $(4, 2)$. What are the coordinates of this point?

(A) $(2, 1)$
(B) $(1, 1)$
(C) $\left(2, \sqrt{2}\right)$
(D) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$
(E) None of the above

---

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

---

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) –2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

---

7. For what value of k will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

---

8. If $h(x)=f^2(x)-g^2(x)$, $f'(x)=-g(x)$, and $g'(x)=f(x)$, then $h'(x)=$

(A) 0 (B) 1 (C) $-4f(x)g(x)$

(D) $(-g(x))^2-(f(x))^2$ (E) $-2(-g(x)+f(x))$

---

9. The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi}\sqrt{3+\cos\theta}\,d\theta$ (B) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$ (C) $2\int_0^{\pi/2}(3+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ (E) $2\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$

---

10. $\int_0^1 \frac{x^2}{x^2+1}\,dx=$

(A) $\frac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\frac{1}{2}\ln 2$ (E) $\frac{4+\pi}{4}$

11. The point <u>on the curve</u> $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$
(B) 0
(C) $-\frac{1}{2}$
(D) -1
(E) none of the above

---

12. If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$
(D) $e^{-x^2} - 1$ (E) e^{-x^2}

---

13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

---

14. If $y = x^2 + 2$ and $u = 2x - 1$, then $\frac{dy}{du} =$

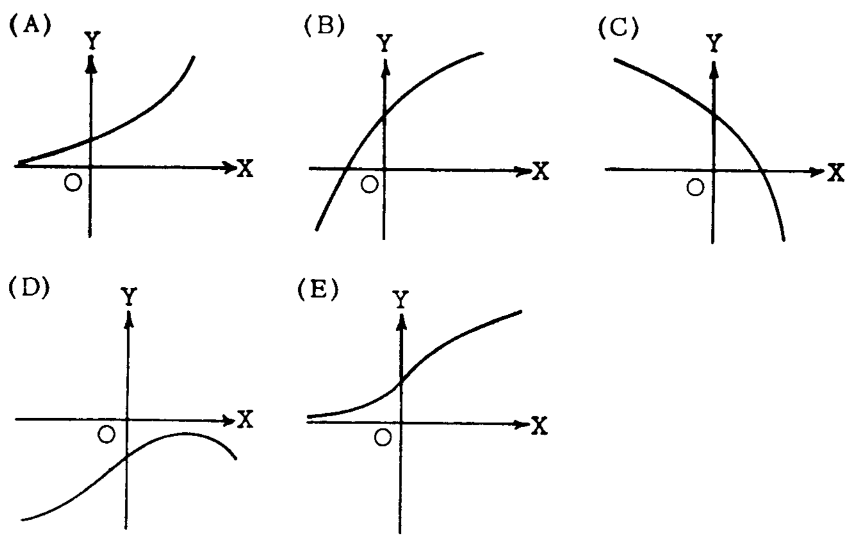
(A) $\frac{2x^2 - 2x + 4}{(2x-1)^2}$ (B) $6x^2 - 2x + 4$ (C) x^2
(D) x (E) $\frac{1}{x}$

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

---

16. If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

---

18. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

20. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$

(D) $y = 0$ (E) $\pi x - 2y = 0$

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

---

22. If $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 2}}\,dt$, which of the following is FALSE?

(A) $f(0) = 0$

(B) f is continuous at x for all $x \geq 0$.

(C) $f(1) > 0$

(D) $f'(1) = \frac{1}{\sqrt{3}}$

(E) $f(-1) > 0$

23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3}+e^{-x}$ (C) $1+e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$

---

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

---

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m .

(B) increases as m increases.

(C) decreases as m increases.

(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.

(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

---

26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

(E) none of the above

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

---

28. What is $\lim_{x\to 0} \frac{e^{2x} - 1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

---

29. $\int_0^1 \left(4 - x^2\right)^{-\frac{3}{2}} dx =$

(A) $\frac{2-\sqrt{3}}{3}$ (B) $\frac{2\sqrt{3}-3}{4}$ (C) $\frac{\sqrt{3}}{12}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{3}}{2}$

---

30. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

---

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^x$

---

32. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x (B) $x < -1$ (C) $x \geq -1$ (D) $x > -1$ (E) All values of x

---

33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

---

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

---

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

---

37. Of the following choices of δ, which is the largest that could be used successfully with an arbitrary ε in an epsilon-delta proof of $\lim_{x \to 2} (1 - 3x) = -5$?

(A) $\delta = 3\varepsilon$ (B) $\delta = \varepsilon$ (C) $\delta = \frac{\varepsilon}{2}$ (D) $\delta = \frac{\varepsilon}{4}$ (E) $\delta = \frac{\varepsilon}{5}$

---

38. If $f(x) = \left(x^2 + 1\right)^{(2-3x)}$, then $f'(1) =$

(A) $-\frac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2}\ln(2)$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$

---

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only
(D) nonzero n, only (E) all n

---

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

---

42. If $\int x^2 \cos x\,dx = f(x) - \int 2x \sin x\,dx$, then $f(x) =$

(A) $2\sin x + 2x\cos x + C$

(B) $x^2 \sin x + C$

(C) $2x\cos x - x^2 \sin x + C$

(D) $4\cos x - 2x\sin x + C$

(E) $\left(2 - x^2\right)\cos x - 4\sin x + C$

---

43. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\,dx$

(B) $\int_a^b \sqrt{x + \tan x}\,dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\,dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\,dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\,dx$

44. If $f''(x) - f'(x) - 2f(x) = 0$, $f'(0) = -2$, and $f(0) = 2$, then $f(1) =$

(A) $e^2 + e^{-1}$ (B) 1 C) 0 (D) e^2 (E) $2e^{-1}$

---

45. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

(A) $0 < x < 2$ (B) $0 \le x \le 2$ (C) $-2 < x \le 0$

(D) $-2 \le x < 0$ (E) $-2 \le x \le 0$

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. $\int \left(x^3 - 3x\right) dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

---

2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g\left(f(x)\right) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

(D) 225 (E) 5

---

3. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

---

4. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

---

5. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

---

6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) –1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

7. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$ (E) $y = \left(x^2+1\right)^2 - 1$

---

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

---

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

---

10. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) –1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

---

11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

---

12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) –6 (B) –3 (C) –1 (D) 2

(E) It cannot be determined from the information given.

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$

(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

---

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

---

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

---

18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

19. Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

(A) The function f is periodic.

(B) The graph of f is symmetric with respect to the y-axis.

(C) The graph of f is concave up.

(D) The function f is a strictly increasing function.

(E) The function f is continuous.

---

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$

(B) $F'(b) - F'(a)$

(C) $F(a) - F(b)$

(D) $F(b) - F(a)$

(E) none of the above

---

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3 - 1}{2}$ (C) $\frac{e^4 - e}{2}$ (D) $e^3 - 1$ (E) $e^4 - e$

---

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$

(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

23. $\lim\limits_{h\to 0} \dfrac{1}{h}\ln\left(\dfrac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\dfrac{1}{2}$ (D) 0 (E) nonexistent

---

24. Let $f(x)=\cos(\arctan x)$. What is the range of f?

(A) $\left\{x \,\middle|\, -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}\right\}$ (B) $\{x \mid 0 < x \le 1\}$ (C) $\{x \mid 0 \le x \le 1\}$

(D) $\{x \mid -1 < x < 1\}$ (E) $\{x \mid -1 \le x \le 1\}$

---

25. $\displaystyle\int_0^{\pi/4} \tan^2 x \, dx =$

(A) $\dfrac{\pi}{4}-1$ (B) $1-\dfrac{\pi}{4}$ (C) $\dfrac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\dfrac{\pi}{4}+1$

---

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \dfrac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

---

27. $\displaystyle\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\, dx =$

(A) $1-\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{1}{2}\ln\dfrac{3}{4}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{6}-1$ (E) $2-\sqrt{3}$

---

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\dfrac{4}{3}$ (C) $\dfrac{5}{3}$ (D) 2 (E) 5

29. Let $f(x) = \left| \sin x - \frac{1}{2} \right|$. The maximum value attained by f is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

---

31. If $\log_a\left(2^a\right) = \frac{a}{4}$, then $a =$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

---

32. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2} + C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right) + C$ (C) $5x - \frac{5}{x} + C$

(D) $5\arctan x + C$ (E) $5\ln\left(1+x^2\right) + C$

---

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'\left(x_0\right)$ exists. Which of the following must necessarily be equal to $f'\left(-x_0\right)$?

(A) $f'\left(x_0\right)$

(B) $-f'\left(x_0\right)$

(C) $\frac{1}{f'\left(x_0\right)}$

(D) $\frac{-1}{f'\left(x_0\right)}$

(E) None of the above

34. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

---

36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n! e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n! e^x$

---

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

---

38. If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5 + c$ (B) 5 (C) $5 - c$ (D) $c - 5$ (E) –5

---

39. The point on the curve $2y = x^2$ nearest to $(4,1)$ is

(A) $(0,0)$ (B) $(2,2)$ (C) $(\sqrt{2},1)$ (D) $(2\sqrt{2},4)$ (E) $(4,8)$

---

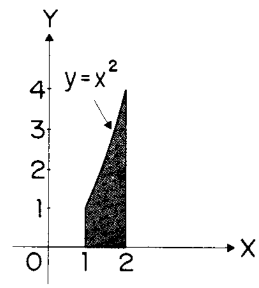
40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

41. Given $f(x)=\begin{cases} x+1 & \text{for } x<0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

---

43. If the solutions of $f(x)=0$ are -1 and 2, then the solutions of $f\left(\frac{x}{2}\right)=0$ are

(A) -1 and 2 (B) $-\frac{1}{2}$ and $\frac{5}{2}$ (C) $-\frac{3}{2}$ and $\frac{3}{2}$

(D) $-\frac{1}{2}$ and 1 (E) -2 and 4

---

44. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4+\frac{h}{32}$ (B) $2+\frac{h}{32}$ (C) $\frac{h}{32}$

(D) $4-\frac{h}{32}$ (E) $2-\frac{h}{32}$

45. If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0)=0$.

(C) $\lim_{x \to x_0} f(x) = f\left(\lim_{x \to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x)=0$ for some $x \in [a,b]$

(E) The graph of f' is a straight line.

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. If $f(x) = e^{1/x}$, then $f'(x) =$

(A) $-\frac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\frac{e^{1/x}}{x}$ (D) $\frac{e^{1/x}}{x^2}$ (E) $\frac{1}{x}e^{(1/x)-1}$

---

2. $\int_0^3 (x+1)^{1/2}\,dx =$

(A) $\frac{21}{2}$ (B) 7 (C) $\frac{16}{3}$ (D) $\frac{14}{3}$ (E) $-\frac{1}{4}$

---

3. If $f(x) = x + \frac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$
(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

---

4. For what non-negative value of b is the line given by $y = -\frac{1}{3}x + b$ normal to the curve $y = x^3$?

(A) 0 (B) 1 (C) $\frac{4}{3}$ (D) $\frac{10}{3}$ (E) $\frac{10\sqrt{3}}{3}$

---

5. $\int_{-1}^{2} \frac{|x|}{x}\,dx$ is

(A) –3 (B) 1 (C) 2 (D) 3 (E) nonexistent

---

6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) –1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

7. If $y = \ln\left(x^2 + y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

---

8. If $y = \sin x$ and $y^{(n)}$ means "the *n*th derivative of *y* with respect to *x*," then the smallest positive integer *n* for which $y^{(n)} = y$ is

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

---

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$
(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

---

10. The length of the curve $y = \ln \sec x$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x\,dx$

(B) $\int_0^b \sec^2 x\,dx$

(C) $\int_0^b (\sec x \tan x)\,dx$

(D) $\int_0^b \sqrt{1 + (\ln \sec x)^2}\,dx$

(E) $\int_0^b \sqrt{1 + \left(\sec^2 x \tan^2 x\right)}\,dx$

---

11. Let $y = x\sqrt{1 + x^2}$. When $x = 0$ and $dx = 2$, the value of dy is

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

---

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

---

16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

(B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$

(C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$

(D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$

(E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

17. The number of bacteria in a culture is growing at a rate of $3{,}000e^{2t/5}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1{,}200e^2$ (B) $3{,}000e^2$ (C) $7{,}500e^2$ (D) $7{,}500e^5$ (E) $\frac{15{,}000}{7}e^7$

---

18. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x \to h} g(x) = g(h)$.

---

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

20. $\int x\sqrt{4-x^2}\,dx =$

(A) $\frac{\left(4-x^2\right)^{3/2}}{3} + C$ (B) $-\left(4-x^2\right)^{3/2} + C$ (C) $\frac{x^2\left(4-x^2\right)^{3/2}}{3} + C$

(D) $-\frac{x^2\left(4-x^2\right)^{3/2}}{3} + C$ (E) $-\frac{\left(4-x^2\right)^{3/2}}{3} + C$

---

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) $e^3 - 1$ (E) $e^4 - e$

22. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$
(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

---

23. $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

---

24. Let $f(x) = 3x + 1$ for all real x and let $\varepsilon > 0$. For which of the following choices of δ is $|f(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$?

(A) $\frac{\varepsilon}{4}$ (B) $\frac{\varepsilon}{2}$ (C) $\frac{\varepsilon}{\varepsilon+1}$ (D) $\frac{\varepsilon+1}{\varepsilon}$ (E) 3ε

---

25. $\int_0^{\pi/4} \tan^2 x \, dx =$

(A) $\frac{\pi}{4} - 1$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\frac{\pi}{4} + 1$

---

26. Which of the following is true about the graph of $y = \ln\left|x^2 - 1\right|$ in the interval $(-1, 1)$?

(A) It is increasing.
(B) It attains a relative minimum at $(0, 0)$.
(C) It has a range of all real numbers.
(D) It is concave down.
(E) It has an asymptote of $x = 0$.

---

27. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \leq x \leq 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ (C) $2\int_0^{\pi/4} \sin^2 y\,dy$

(D) $\int_0^{\pi/4} \sin^2 y\,dy$ (E) $2\int_0^{\pi/6} \sin^2 y\,dy$

---

29. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

(A) $\frac{e}{2}\left(e^2+1\right)$ (B) e (C) $\frac{e^3}{2}$ (D) $\frac{e}{2}$ (E) $\frac{\left(e^3-e\right)}{2}$

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

---

31. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x\ln x}$

---

32. If $y = x^{\ln x}$, then y' is

(A) $\frac{x^{\ln x}\ln x}{x^2}$

(B) $x^{1/x}\ln x$

(C) $\frac{2x^{\ln x}\ln x}{x}$

(D) $\frac{x^{\ln x}\ln x}{x}$

(E) None of the above

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $-\dfrac{1}{f'(x_0)}$

(E) None of the above

---

34. The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

---

36. $\int_0^1 \frac{x+1}{x^2+2x-3}\,dx$ is

(A) $-\ln\sqrt{3}$ (B) $-\frac{\ln\sqrt{3}}{2}$ (C) $\frac{1-\ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) divergent

---

37. $\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) –2 (B) 0 (C) 1 (D) 2 (E) 4

---

38. If $\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) –5

39. Let f and g be differentiable functions such that

$f(1) = 2$, $f'(1) = 3$, $f'(2) = -4$,

$g(1) = 2$, $g'(1) = -3$, $g'(2) = 5$.

If $h(x) = f\left(g(x)\right)$, then $h'(1) =$

(A) –9 (B) –4 (C) 0 (D) 12 (E) 15

---

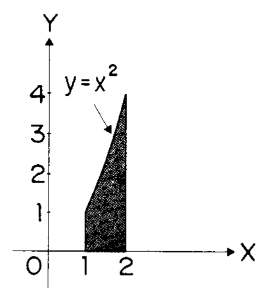
40. The area of the region enclosed by the polar curve $r = 1 - \cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

---

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

43. $\int \arcsin x \, dx =$

(A) $\sin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

(B) $\frac{(\arcsin x)^2}{2} + C$

(C) $\arcsin x + \int \frac{dx}{\sqrt{1-x^2}}$

(D) $x \arccos x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

(E) $x \arcsin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$

---

44. If f is the solution of $x f'(x) - f(x) = x$ such that $f(-1) = 1$, then $f\left(e^{-1}\right) =$

(A) $-2e^{-1}$ (B) 0 C) e^{-1} (D) $-e^{-1}$ (E) $2e^{-2}$

---

45. Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t\,g'(t)\,dt$ for all $x \geq 0$. Which of the following statements is FALSE?

(A) F takes on negative values.

(B) F is continuous for all $x > 0$.

(C) $F(x) = x\,g(x) - \int_0^x g(t)\,dt$

(D) $F'(x)$ exists for all $x > 0$.

(E) F is an increasing function.

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

---

2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

---

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

---

4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x) + C$ (B) $-\frac{1}{2}\cos^2(2x) + C$ (C) $\frac{1}{2}\sin(2x) + C$

(D) $\frac{1}{2}\sin^2(2x) + C$ (E) $-\frac{1}{2}\sin(2x) + C$

---

5. $\lim\limits_{n\to\infty} \frac{4n^2}{n^2 + 10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

6. If $f(x) = x$, then $f'(5) =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

---

7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\, dt$ (D) $\int_1^4 \ln x\, dx$ (E) $\int_1^4 \frac{1}{t}\, dt$

---

8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

---

9. If $\int_{-1}^{1} e^{-x^2}\, dx = k$, then $\int_{-1}^{0} e^{-x^2}\, dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

---

10. If $y = 10^{\left(x^2-1\right)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

---

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

---

12. If $f(g(x)) = \ln\left(x^2+4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

---

14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

---

15. The domain of the function defined by $f(x) = \ln\left(x^2 - 4\right)$ is the set of all real numbers x such that

(A) $|x| < 2$ (B) $|x| \le 2$ (C) $|x| > 2$ (D) $|x| \ge 2$ (E) x is a real number

---

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

---

17. $\int_0^1 xe^{-x}dx =$

(A) $1-2e$ (B) -1 (C) $1-2e^{-1}$ (D) 1 (E) $2e-1$

---

18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

---

19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x) = x+1$ (B) $f(x) = 2x$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

(A) $\frac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2 + 1}$ (E) $\frac{1}{1+\cos^2 x}$

---

21. If the domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of f?

(A) $\{x : -\infty < x < -1\}$ (B) $\{x : -\infty < x < 0\}$ (C) $\{x : -\infty < x < 1\}$

(D) $\{x : -1 < x < \infty\}$ (E) $\{x : 0 < x < \infty\}$

---

22. $\int_1^2 \frac{x^2-1}{x+1}\, dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$

---

23. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

(A) −6 (B) −4 (C) 0 (D) 2 (E) 6

---

24. If $\int_{-2}^{2} \left(x^7 + k\right) dx = 16$, then $k =$

(A) −12 (B) −4 (C) 0 (D) 4 (E) 12

---

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$

26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

---

27. $\int_0^3 |x-1|\, dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

---

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) −45 (B) −30 (C) −15 (D) −10 (E) −5

---

29. Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

---

30. $\int \tan(2x)\, dx =$

(A) $-2\ln|\cos(2x)| + C$ (B) $-\frac{1}{2}\ln|\cos(2x)| + C$ (C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

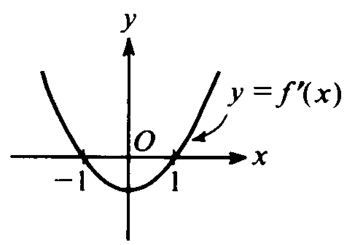
31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

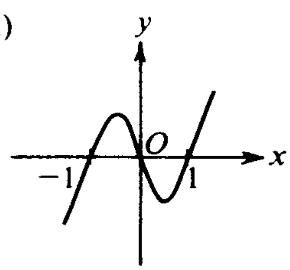
---

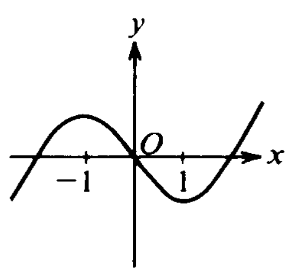
32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

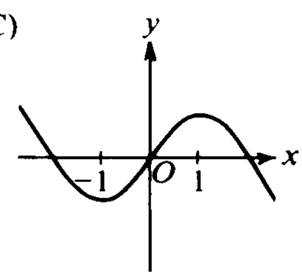
(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

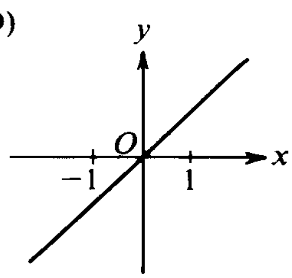


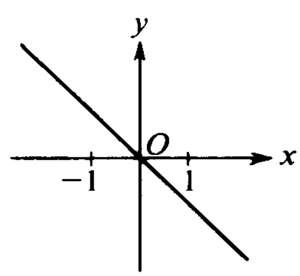
33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


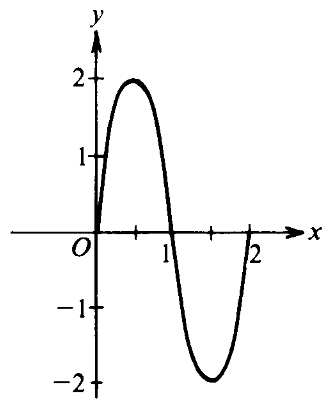
(C)


(D)


(E)


34. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$



35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

---

36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

37. $\lim_{x\to 0}(x\csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

I. $f'(x) \le g'(x)$ for all real x

II. $f''(x) \le g''(x)$ for all real x

III. $\int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

---

39. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

---

40. Let f be a continuous function on the closed interval $[0, 2]$. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

---

41. If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above

42. $\frac{d}{dx}\int_{2}^{x}\sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

---

43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

---

44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0,2]$ is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

---

45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the <u>y-axis</u>. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

---

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

(A) –1 only (B) 0 only (C) 1 only (D) –1 and 1 only (E) –1, 0 and 1

---

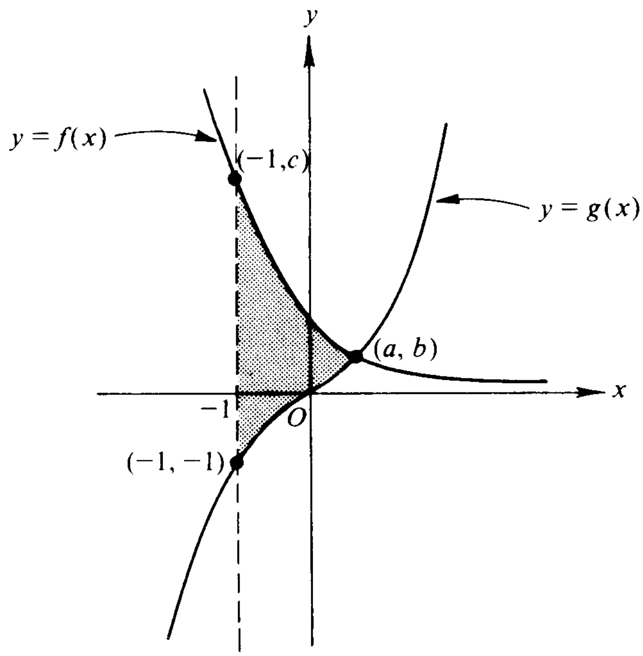
3. $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

---

4. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is

(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$



5. The curves $y=f(x)$ and $y=g(x)$ shown in the figure above intersect at the point (a,b). The area of the shaded region enclosed by these curves and the line $x=-1$ is given by

(A) $\int_0^a (f(x)-g(x))\,dx+\int_{-1}^0 (f(x)+g(x))\,dx$

(B) $\int_{-1}^b g(x)\,dx+\int_b^c f(x)\,dx$

(C) $\int_{-1}^c (f(x)-g(x))\,dx$

(D) $\int_{-1}^a (f(x)-g(x))\,dx$

(E) $\int_{-1}^a (|f(x)|-|g(x)|)\,dx$

---

6. If $f(x)=\dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right)=$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1+\dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2}-1$ (E) $1-\dfrac{\pi}{2}$

7. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin \frac{x}{5} + C$ (B) $\arcsin x + C$ (C) $\frac{1}{5}\arcsin \frac{x}{5} + C$

(D) $\sqrt{25-x^2} + C$ (E) $2\sqrt{25-x^2} + C$

---

8. If f is a function such that $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist.

(B) f is not defined at $x = 2$.

(C) The derivative of f at $x = 2$ is 0.

(D) f is continuous at $x = 0$.

(E) $f(2) = 0$

---

9. If $xy^2 + 2xy = 8$, then, at the point $(1, 2)$, y' is

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

---

10. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

(A) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

(B) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$

(C) $\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$

(D) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$

(E) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

11. $\dfrac{d}{dx}\ln\left(\dfrac{1}{1-x}\right) =$

(A) $\dfrac{1}{1-x}$ (B) $\dfrac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

---

12. $\displaystyle\int \frac{dx}{(x-1)(x+2)} =$

(A) $\dfrac{1}{3}\ln\left|\dfrac{x-1}{x+2}\right| + C$ (B) $\dfrac{1}{3}\ln\left|\dfrac{x+2}{x-1}\right| + C$ (C) $\dfrac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

(D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right) + C$ (E) $\ln\left|(x-1)(x+2)^2\right| + C$

---

13. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

---

14. Which of the following series are convergent?

I. $1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots + \dfrac{1}{n^2} + \ldots$

II. $1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots + \dfrac{1}{n} + \ldots$

III. $1 - \dfrac{1}{3} + \dfrac{1}{3^2} - \ldots + \dfrac{(-1)^{n+1}}{3^{n-1}} + \ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

---

15. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

(A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

16. Which of the following functions shows that the statement "If a function is continuous at $x = 0$, then it is differentiable at $x = 0$" is false?

(A) $f(x) = x^{-\frac{4}{3}}$ (B) $f(x) = x^{-\frac{1}{3}}$ (C) $f(x) = x^{\frac{1}{3}}$ (D) $f(x) = x^{\frac{4}{3}}$ (E) $f(x) = x^3$

---

17. If $f(x) = x\ln\left(x^2\right)$, then $f'(x) =$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

---

18. $\int \sin(2x+3)\,dx =$

(A) $-2\cos(2x+3)+C$ (B) $-\cos(2x+3)+C$ (C) $-\frac{1}{2}\cos(2x+3)+C$

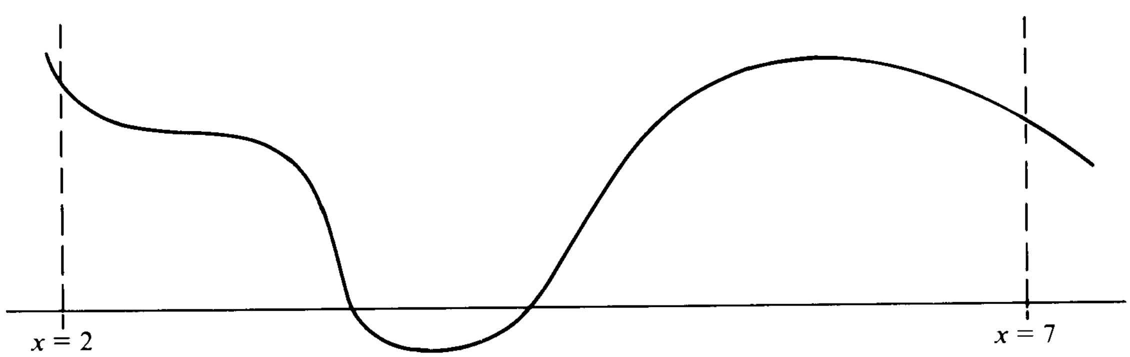
(D) $\frac{1}{2}\cos(2x+3)+C$ (E) $\cos(2x+3)+C$

---

19. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x)+f''(x)$ (B) $f'(x)+\left(f''(x)\right)^2$ (C) $\left(f'(x)+f''(x)\right)^2$

(D) $\left(f'(x)\right)^2+f''(x)$ (E) $2f'(x)+f''(x)$



20. The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

21. If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

---

22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

---

23. $\lim\limits_{h\to 0} \dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h}$ is

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

---

24. The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \le \theta \le \dfrac{\pi}{2}$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) $\dfrac{\pi}{8}$ (E) $\dfrac{\pi}{4}$

---

25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

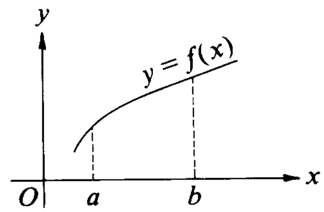
(A) No values (B) 0 only (C) $\dfrac{1}{2}$ only (D) 1 only (E) 0 and $\dfrac{1}{2}$

---

26. For $0 < x < \dfrac{\pi}{2}$, if $y = (\sin x)^x$, then $\dfrac{dy}{dx}$ is

(A) $x\ln(\sin x)$ (B) $(\sin x)^x \cot x$ (C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x + \sin x)$ (E) $(\sin x)^x(x\cot x + \ln(\sin x))$



27. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

---

28. An antiderivative of $f(x) = e^{x+e^x}$ is

(A) $\dfrac{e^{x+e^x}}{1+e^x}$ (B) $\left(1+e^x\right)e^{x+e^x}$ (C) e^{1+e^x} (D) e^{x+e^x} (E) e^{e^x}

---

29. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$ is

(A) 0 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\pi}{4}$ (D) 1 (E) nonexistent

---

30. If $x = t^3 - t$ and $y = \sqrt{3t+1}$, then $\dfrac{dy}{dx}$ at $t = 1$ is

(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{8}{3}$ (E) 8

---

31. What are all values of x for which the series $\sum_{n=1}^{\infty} \dfrac{(x-1)^n}{n}$ converges?

(A) $-1 \le x < 1$ (B) $-1 \le x \le 1$ (C) $0 < x < 2$ (D) $0 \le x < 2$ (E) $0 \le x \le 2$

32. An equation of the line <u>normal</u> to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

(A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

---

33. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

---

34. Which of the following gives the area of the surface generated by revolving about the y-axis the arc of $x = y^3$ from $y = 0$ to $y = 1$?

(A) $2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$

(B) $2\pi\int_0^1 y^3\sqrt{1+y^6}\,dy$

(C) $2\pi\int_0^1 y^3\sqrt{1+3y^2}\,dy$

(D) $2\pi\int_0^1 y\sqrt{1+9y^4}\,dy$

(E) $2\pi\int_0^1 y\sqrt{1+y^6}\,dy$

---

35. The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

(B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$

(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9-y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3 + \sqrt{9-y}\right)^2 dy$

36. $\int_{-1}^{1} \frac{3}{x^2}\,dx$ is

(A) –6 (B) –3 (C) 0 (D) 6 (E) nonexistent

---

37. The general solution for the equation $\frac{dy}{dx} + y = xe^{-x}$ is

(A) $y = \frac{x^2}{2}e^{-x} + Ce^{-x}$

(B) $y = \frac{x^2}{2}e^{-x} + e^{-x} + C$

(C) $y = -e^{-x} + \frac{C}{1+x}$

(D) $y = xe^{-x} + Ce^{-x}$

(E) $y = C_1e^{x} + C_2xe^{-x}$

---

38. $\lim_{x\to\infty}\left(1+5e^{x}\right)^{\frac{1}{x}}$ is

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent

---

39. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

---

40. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_{2}^{4} \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

(A) $\int_{1}^{2} \frac{1-u^2}{u}\,du$

(B) $\int_{2}^{4} \frac{1-u^2}{u}\,du$

(C) $\int_{1}^{2} \frac{1-u^2}{2u}\,du$

(D) $\int_{1}^{2} \frac{1-u^2}{4u}\,du$

(E) $\int_{2}^{4} \frac{1-u^2}{2u}\,du$

41. What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$ (B) 4 (C) $\frac{14}{3}$ (D) $\frac{16}{3}$ (E) 7

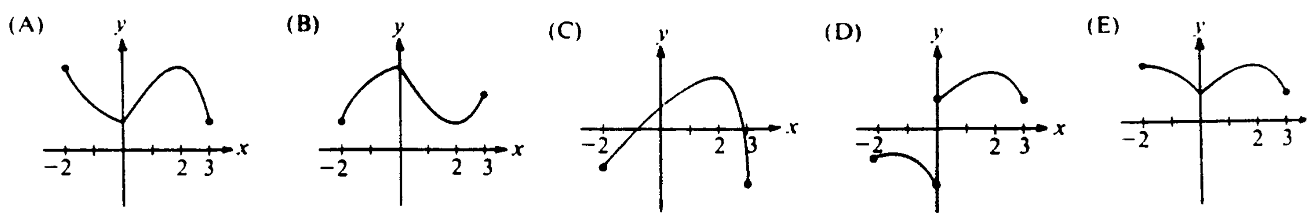
---

42. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

---

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



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44. At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

---

45. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{3n}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x^2}\,dx$ (B) $3\int_0^1 \left(\frac{1}{x}\right)^2 dx$ (C) $\int_0^3 \left(\frac{1}{x}\right)^2 dx$

(D) $\int_0^3 x^2\,dx$ (E) $3\int_0^3 x^2\,dx$

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x + 2e^x\right)$ (C) $xe^x(x+2)$

(D) $2x + e^x$ (E) $2x + e$

---

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x : x \neq 3\}$ (B) $\{x : |x| \leq 2\}$ (C) $\{x : |x| \geq 2\}$

(D) $\{x : |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x : x \geq 2 \text{ and } x \neq 3\}$

---

3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

---

4. The graph of $y = \frac{-5}{x - 2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

---

5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

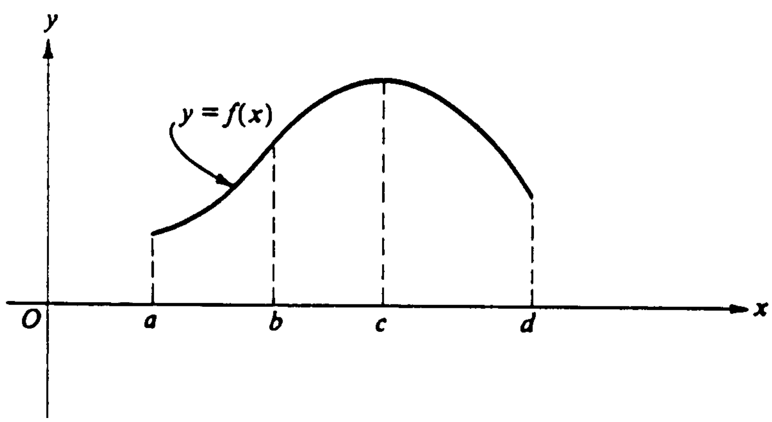
(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

---

7. $\int \frac{x\,dx}{\sqrt{3x^2 + 5}} =$

(A) $\frac{1}{9}\left(3x^2 + 5\right)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}\left(3x^2 + 5\right)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}\left(3x^2 + 5\right)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}\left(3x^2 + 5\right)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}\left(3x^2 + 5\right)^{\frac{1}{2}} + C$



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

9. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

---

10. If $\int_0^k \left(2kx - x^2\right) dx = 18$, then $k =$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

---

11. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

---

13. If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

---

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

15. If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

---

16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

---

17. $\int_0^1 (3x-2)^2 \, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

---

18. If $y = 2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

---

19. $\int_2^3 \frac{x}{x^2+1} \, dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

---

20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b?

I. $f(x) = 0$
II. $f'(x) = 0$
III. $f''(x) = 0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

---

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

---

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

---

24. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

---

25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\,dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

---

26. $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

---

28. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

---

29. The $\lim\limits_{h \to 0} \dfrac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

---

30. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the $\underline{y\text{-axis}}$, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi \int_0^1 xe^{2x}\, dx$

(B) $2\pi \int_0^1 e^{2x}\, dx$

(C) $\pi \int_0^1 e^{4x}\, dx$

(D) $\pi \int_0^e y \ln y\, dy$

(E) $\frac{\pi}{4} \int_0^e \ln^2 y\, dy$

31. If $f(x)=\dfrac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x)=$

(A) $\dfrac{x-1}{x}$ (B) $\dfrac{x+1}{x}$ (C) $\dfrac{x}{1-x}$ (D) $\dfrac{x}{x+1}$ (E) x

---

32. Which of the following does NOT have a period of π ?

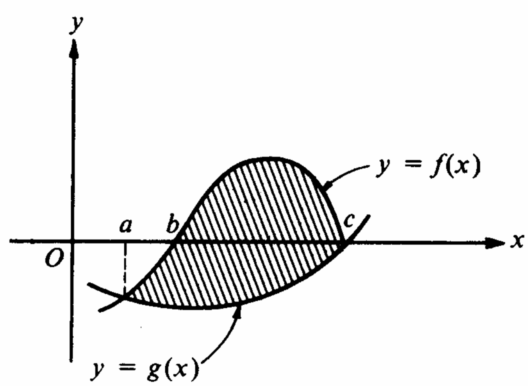
(A) $f(x)=\sin\left(\dfrac{1}{2}x\right)$ (B) $f(x)=\left|\sin x\right|$ (C) $f(x)=\sin^2 x$

(D) $f(x)=\tan x$ (E) $f(x)=\tan^2 x$

---

33. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) –2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c \left(\left|f(x)\right|-\left|g(x)\right|\right)dx$

(B) $\int_b^c f(x)\,dx-\int_a^c g(x)\,dx$

(C) $\int_a^c \left(g(x)-f(x)\right)dx$

(D) $\int_a^c \left(f(x)-g(x)\right)dx$

(E) $\int_a^b \left(g(x)-f(x)\right)dx+\int_b^c \left(f(x)-g(x)\right)dx$

35. $4\cos\left(x+\frac{\pi}{3}\right)=$

(A) $2\sqrt{3}\cos x-2\sin x$ (B) $2\cos x-2\sqrt{3}\sin x$ (C) $2\cos x+2\sqrt{3}\sin x$

(D) $2\sqrt{3}\cos x+2\sin x$ (E) $4\cos x+2$

---

36. What is the average value of y for the part of the curve $y=3x-x^2$ which is in the first quadrant?

(A) –6 (B) –2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

---

37. If $f(x)=e^x\sin x$, then the number of zeros of f on the closed interval $[0,2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

38. For $x>0$, $\int\left(\frac{1}{x}\int_1^x\frac{du}{u}\right)dx=$

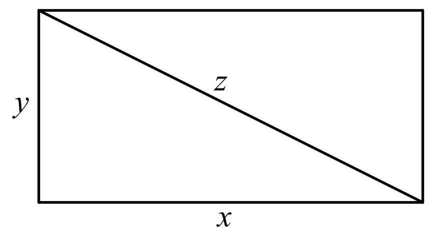
(A) $\frac{1}{x^3}+C$ (B) $\frac{8}{x^4}-\frac{2}{x^2}+C$ (C) $\ln(\ln x)+C$

(D) $\frac{\ln\left(x^2\right)}{2}+C$ (E) $\frac{(\ln x)^2}{2}+C$

---

39. If $\int_1^{10}f(x)\,dx=4$ and $\int_{10}^{3}f(x)\,dx=7$, then $\int_1^{3}f(x)\,dx=$

(A) –3 (B) 0 (C) 3 (D) 10 (E) 11



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

---

41. If $\lim_{x\to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

---

42. The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \frac{x}{x+1}$ (D) $y = \frac{x^2}{x-1}$ (E) $y = e^{-x}$

---

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

45. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$
(B) $2\sqrt{2}$
(C) $2\sqrt[3]{4}$
(D) 4
(E) 8

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the x-axis is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) 1

---

2. $\int_0^1 x\left(x^2+2\right)^2 \, dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

---

3. If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

---

4. If u, v, and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

(A) $\frac{uv' + u'v}{w'}$ (B) $\frac{u'v'w - uvw'}{w^2}$ (C) $\frac{uvw' - uv'w - u'vw}{w^2}$

(D) $\frac{u'vw + uv'w + uvw'}{w^2}$ (E) $\frac{uv'w + u'vw - uvw'}{w^2}$

5. Let f be the function defined by the following.

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

---

6. If $y^2-2xy=16$, then $\frac{dy}{dx}=$

(A) $\frac{x}{y-x}$ (B) $\frac{y}{x-y}$ (C) $\frac{y}{y-x}$ (D) $\frac{y}{2y-x}$ (E) $\frac{2y}{x-y}$

---

7. $\int_2^{+\infty}\frac{dx}{x^2}$ is

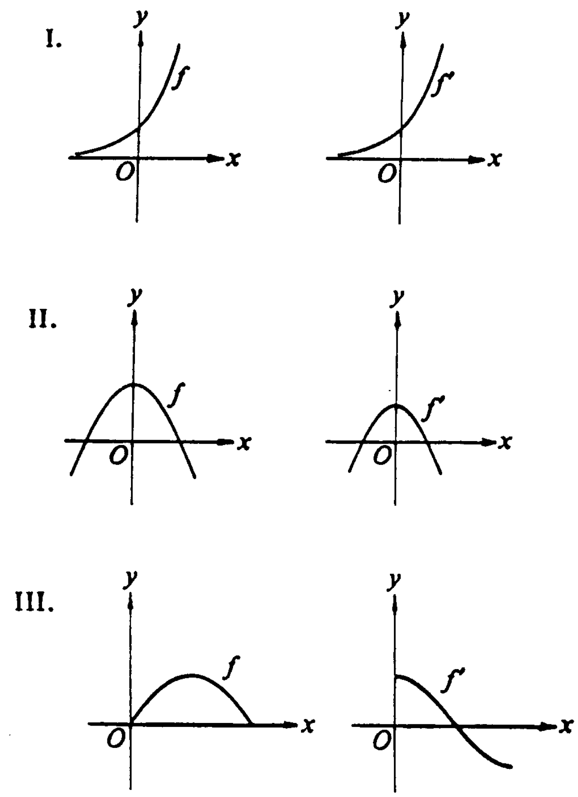
(A) $\frac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) nonexistent

---

8. If $f(x)=e^x$, then $\ln\left(f'(2)\right)=$

(A) 2 (B) 0 (C) $\frac{1}{e^2}$ (D) $2e$ (E) e^2

9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



(A) I only (B) II only (C) III only (D) I and III (E) II and III

---

10. $\lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin x}{h}$ is

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

---

11. If $x + 7y = 29$ is an equation of the line <u>normal</u> to the graph of f at the point $(1,4)$, then $f'(1) =$

(A) 7 (B) $\dfrac{1}{7}$ (C) $-\dfrac{1}{7}$ (D) $-\dfrac{7}{29}$ (E) -7

---

12. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

(A) 20 m (B) 14 m (C) 7 m (D) 6 m (E) 3 m

13. $\sin(2x) =$

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \ldots$

(B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1} (2x)^{2n-1}}{(2n-1)!} + \ldots$

(C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n (2x)^{2n}}{(2n)!} + \ldots$

(D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

(E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

---

14. If $F(x) = \int_1^{x^2} \sqrt{1+t^3}\, dt$, then $F'(x) =$

(A) $2x\sqrt{1+x^6}$

(B) $2x\sqrt{1+x^3}$

(C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$

(E) $\int_1^{x^2} \frac{3t^2}{2\sqrt{1+t^3}}\, dt$

---

15. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is

(A) $\left(2t, \frac{2}{(2t+3)}\right)$

(B) $\left(2t, \frac{-4}{(2t+3)^2}\right)$

(C) $\left(2, \frac{4}{(2t+3)^2}\right)$

(D) $\left(2, \frac{2}{(2t+3)^2}\right)$

(E) $\left(2, \frac{-4}{(2t+3)^2}\right)$

16. $\int xe^{2x}dx =$

(A) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$ (B) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{2} + C$ (C) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{2} + C$ (E) $\frac{x^2e^{2x}}{4} + C$

---

17. $\int_2^3 \frac{3}{(x-1)(x+2)}dx =$

(A) $-\frac{33}{20}$ (B) $-\frac{9}{20}$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln\left(\frac{8}{5}\right)$ (E) $\ln\left(\frac{2}{5}\right)$

---

18. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}dx$?

(A) $e^2 + e^0 + e^{-2}$ (B) $e^4 + e^2 + e^0$ (C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$ (E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

---

19. A polynomial $p(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5,7)$ and no other critical points. How many zeros does $p(x)$ have?

(A) One (B) Two (C) Three (D) Four (E) Five

---

20. The statement "$\lim_{x\to a} f(x) = L$" means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that

(A) if $0 < |x-a| < \varepsilon$, then $|f(x) - L| < \delta$

(B) if $0 < |f(x) - L| < \varepsilon$, then $|x-a| < \delta$

(C) if $|f(x) - L| < \delta$, then $0 < |x-a| < \varepsilon$

(D) $0 < |x-a| < \delta$ and $|f(x) - L| < \varepsilon$

(E) if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$

21. The average value of $\frac{1}{x}$ on the closed interval $[1,3]$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\ln 3}{2}$ (E) $\ln 3$

---

22. If $f(x) = \left(x^2+1\right)^x$, then $f'(x) =$

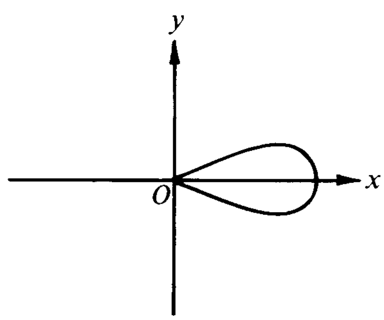
(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2+1\right) + \frac{2x^2}{x^2+1}$

(E) $\left(x^2+1\right)^x\left[\ln\left(x^2+1\right) + \frac{2x^2}{x^2+1}\right]$



23. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos(3\theta)\,d\theta$ (B) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos(3\theta)\,d\theta$ (C) $8\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2(3\theta)\,d\theta$

(D) $16\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$ (E) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$

24. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then $c =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{4}{3}$ (E) 2

---

25. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$

---

26. If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

(A) $\int_{-1}^{1} f(x)\,dx > 0$

(B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

(C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

(D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

(E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

---

27. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

(A) –3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

28. $\frac{d}{dx}\ln\left|\cos\left(\frac{\pi}{x}\right)\right|$ is

(A) $\frac{-\pi}{x^2\cos\left(\frac{\pi}{x}\right)}$ (B) $-\tan\left(\frac{\pi}{x}\right)$ (C) $\frac{1}{\cos\left(\frac{\pi}{x}\right)}$

(D) $\frac{\pi}{x}\tan\left(\frac{\pi}{x}\right)$ (E) $\frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$

---

29. The region R in the first quadrant is enclosed by the lines $x=0$ and $y=5$ and the graph of $y=x^2+1$. The volume of the solid generated when R is revolved about the <u>y-axis</u> is

(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$

---

30. $\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^i =$

(A) $\frac{3}{2}-\left(\frac{1}{3}\right)^n$ (B) $\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^n\right]$ (C) $\frac{3}{2}\left(\frac{1}{3}\right)^n$

(D) $\frac{2}{3}\left(\frac{1}{3}\right)^n$ (E) $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

---

31. $\int_0^2 \sqrt{4-x^2}\,dx =$

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) π (D) 2π (E) 4π

---

32. The general solution of the differential equation $y' = y + x^2$ is $y =$

(A) Ce^x (B) $Ce^x + x^2$ (C) $-x^2-2x-2+C$

(D) $e^x - x^2 - 2x - 2 + C$ (E) $Ce^x - x^2 - 2x - 2$

33. The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_0^2 \sqrt{1+x^6}\,dx$

(B) $\int_0^2 \sqrt{1+3x^2}\,dx$

(C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

(D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$

(E) $\int_0^2 \sqrt{1+9x^4}\,dx$

---

34. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x$

(B) $y = 8x$

(C) $y = 2x - 1$

(D) $y = 4x - 5$

(E) $y = 8x + 13$

---

35. If k is a positive integer, then $\lim\limits_{x\to+\infty} \dfrac{x^k}{e^x}$ is

(A) 0

(B) 1

(C) e

(D) $k!$

(E) nonexistent

---

36. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \dfrac{\pi}{2}$. The volume of the solid obtained by revolving R about the x-axis is given by

(A) $2\pi\int_0^{\frac{\pi}{2}} x\sin x\,dx$

(B) $2\pi\int_0^{\frac{\pi}{2}} x\cos x\,dx$

(C) $\pi\int_0^{\frac{\pi}{2}} (1-\sin x)^2\,dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$

(E) $\pi\int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)dx$

---

37. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\dfrac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\dfrac{4}{27}$

(B) $\dfrac{4}{9}$

(C) $\dfrac{3}{4}$

(D) $\dfrac{4}{3}$

(E) $\dfrac{16}{9}$

38. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

(A) $-1 \le x \le 1$ (B) $-1 < x \le 1$ (C) $-1 \le x < 1$

(D) $-1 < x < 1$ (E) All real x

---

39. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$

(D) $\tan x + 5$ (E) $\tan x + 5e^x$

---

40. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

---

41. $\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \ldots + \sqrt{\frac{n}{n}}\right] =$

(A) $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (B) $\int_0^1 \sqrt{x}\,dx$ (C) $\int_0^1 x\,dx$

(D) $\int_1^2 x\,dx$ (E) $2\int_1^2 x\sqrt{x}\,dx$

---

42. If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

(A) 6 (B) 3 (C) 0 (D) -1 (E) -6

43. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

---

44. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

---

45. What is the area of the largest rectangle that can be inscribed in the ellipse $4x^2 + 9y^2 = 36$?

(A) $6\sqrt{2}$ (B) 12 (C) 24 (D) $24\sqrt{2}$ (E) 36

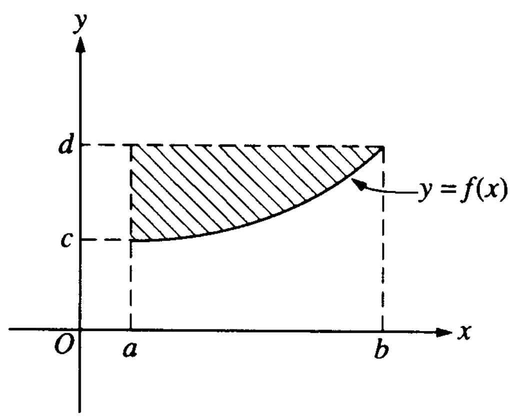
90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) –6 (B) –3 (C) 3 (D) 6 (E) 8



2. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$ (B) $\int_a^b (d - f(x))dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)[f(b) - f(a)]$ (E) $(d-c)[f(b) - f(a)]$

---

3. $\lim_{n\to\infty} \dfrac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) –5 (B) –2 (C) 1 (D) 3 (E) nonexistent

4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$

(B) $-\frac{x^2 + y}{x + y^2}$

(C) $-\frac{x^2 + y}{x + 2y}$

(D) $-\frac{x^2 + y}{2y^2}$

(E) $\frac{-x^2}{1 + 2y^2}$

---

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4 (B) -2 (C) -1 (D) 0 (E) 2

---

6. The area of the region enclosed by the curve $y = \frac{1}{x - 1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln \frac{2}{3}$ (C) $\ln \frac{4}{3}$ (D) $\ln \frac{3}{2}$ (E) $\ln 6$

---

7. An equation of the line tangent to the graph of $y = \frac{2x + 3}{3x - 2}$ at the point $(1, 5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

8. If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

---

9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

---

10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

---

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

---

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\int_a^b \left(f(x) + g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

13. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

---

14. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

---

15. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

---

16. The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

17. $\int (x^2+1)^2\,dx =$

(A) $\dfrac{(x^2+1)^3}{3}+C$

(B) $\dfrac{(x^2+1)^3}{6x}+C$

(C) $\left(\dfrac{x^3}{3}+x\right)^2+C$

(D) $\dfrac{2x(x^2+1)^3}{3}+C$

(E) $\dfrac{x^5}{5}+\dfrac{2x^3}{3}+x+C$

---

18. If $f(x)=\sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2}<x<\dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) π (E) $\dfrac{3\pi}{2}$

---

19. Let f be the function defined by $f(x)=\begin{cases} x^3 & \text{for } x\le 0, \\ x & \text{for } x>0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x=0$.

(C) f has a relative maximum.

(D) $f'(0)=0$

(E) $f'(x)>0$ for $x\neq 0$

20. Let R be the region in the first quadrant enclosed by the graph of $y=(x+1)^{\frac{1}{3}}$, the line $x=7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

(A) $\pi\int_0^7 (x+1)^{\frac{2}{3}}\,dx$ (B) $2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\,dx$ (C) $\pi\int_0^2 (x+1)^{\frac{2}{3}}\,dx$

(D) $2\pi\int_0^2 x(x+1)^{\frac{1}{3}}\,dx$ (E) $\pi\int_0^7 (y^3-1)^2\,dy$

---

21. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

---

22. An antiderivative for $\frac{1}{x^2-2x+2}$ is

(A) $-(x^2-2x+2)^{-2}$

(B) $\ln(x^2-2x+2)$

(C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

(E) $\arctan(x-1)$

---

23. How many critical points does the function $f(x)=(x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

---

24. If $f(x)=(x^2-2x-1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

25. $\frac{d}{dx}\left(2^x\right) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

---

26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

---

27. The function f given by $f(x) = x^3 + 12x - 24$ is

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

---

28. $\int_1^{500}\left(13^x - 11^x\right)dx + \int_2^{500}\left(11^x - 13^x\right)dx =$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

---

29. $\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

---

30. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\,\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

31. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

---

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

---

33. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

---

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

---

35. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

(A) –5 (B) –1 (C) 0 (D) 1 (E) 5

36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

---

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

II. $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

III. $\lim\limits_{x\to a}\dfrac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\dfrac{x^3}{3} + \cos x - x + 1$

(B) $\dfrac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

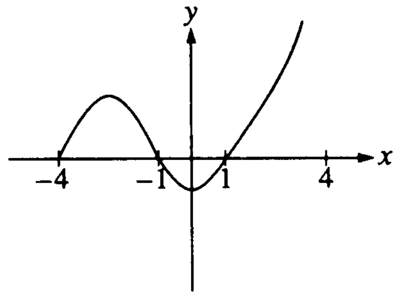
(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

---

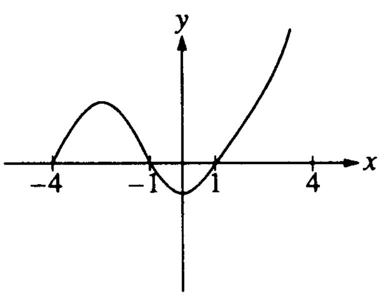
39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{\pi}$ (D) 1 (E) 2

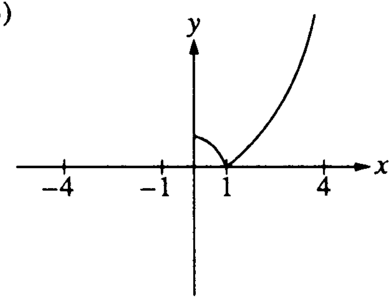


40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

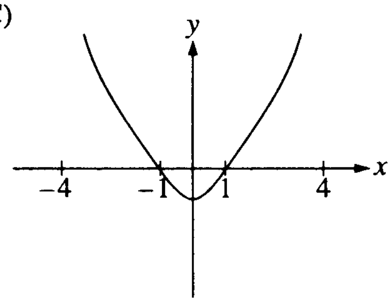
(A)



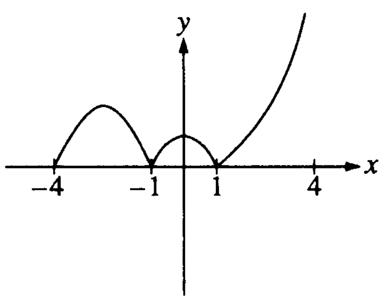
(B)



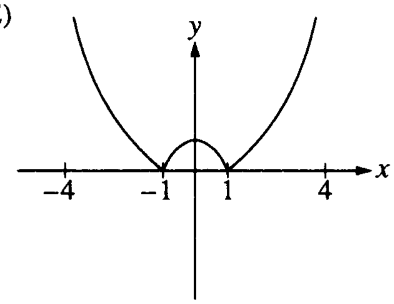
(C)



(D)



(E)



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41. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

---

42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

43. $\int x\,f(x)\,dx =$

(A) $x\,f(x) - \int x\,f'(x)\,dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)dx$

(C) $x\,f(x) - \frac{x^2}{2} f(x) + C$

(D) $x\,f(x) - \int f'(x)\,dx$

(E) $\frac{x^2}{2} \int f(x)\,dx$

---

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

---

45. If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a third approximation of $x_3 =$

(A) 0.682 (B) 0.686 (C) 0.694 (D) 0.750 (E) 1.637

90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

---

2. If $f(x) = 2x^2 + 1$, then $\lim_{x \to 0} \frac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

---

3. If p is a polynomial of degree n, $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t)dt$?

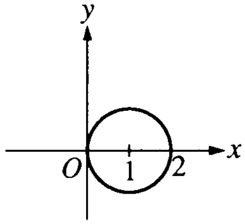
(A) 0 (B) 1 (C) $n-1$ (D) n (E) $n+1$

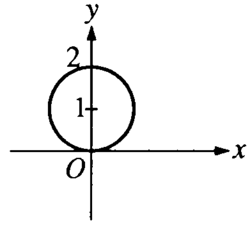
---

4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

(A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

5. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

(A)


(B)


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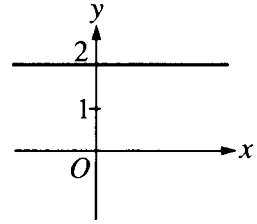

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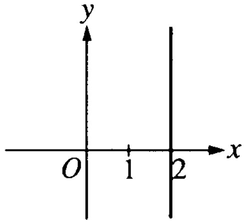
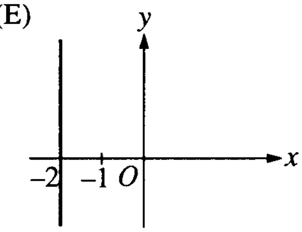


(E)


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6. If $x = t^2 + 1$ and $y = t^3$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{3}{4t}$ (B) $\dfrac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\dfrac{3}{2}$

---

7. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\dfrac{1}{4}(e-1)$ (B) $\dfrac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

---

8. If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

9. If $f(x)=1+x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.

(B) f has a minimum at $x=0$.

(C) f is increasing for $x>0$.

(D) $f'(x)$ exists for all x.

(E) $f''(x)$ is negative for $x>0$.

---

10. Which of the following functions are continuous at $x=1$?

I. $\ln x$

II. e^x

III. $\ln(e^x-1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

11. $\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ is

(A) $7^{\frac{2}{3}}$ (B) $\frac{3}{2}\left(7^{\frac{2}{3}}\right)$ (C) $9^{\frac{2}{3}}+7^{\frac{2}{3}}$ (D) $\frac{3}{2}\left(9^{\frac{2}{3}}+7^{\frac{2}{3}}\right)$ (E) nonexistent

---

12. The position of a particle moving along the x-axis is $x(t)=\sin(2t)-\cos(3t)$ for time $t\geq 0$. When $t=\pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) -9

---

13. If $\frac{dy}{dx}=x^2y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}}+7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3}+1$

14. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

(A) None (B) One (C) Two (D) Three (E) Four

---

15. If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2 \tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2 \tan x\, e^{\tan^2 x}$

---

16. Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None (B) II only (C) III only (D) I and III (E) II and III

---

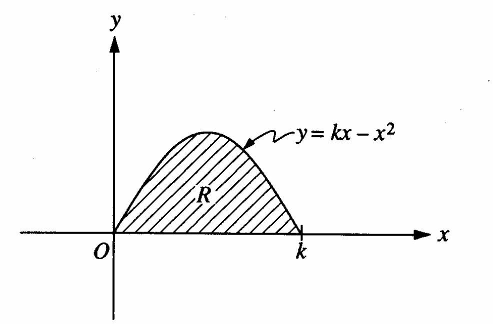
17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

---

18. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$



19. The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k ?

(A) 1.51 (B) 2.09 (C) 2.49 (D) 4.18 (E) 4.77

---

20. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

---

21. The value of the derivative of $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt[4]{2x + 1}}$ at $x = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

22. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

---

23. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

(A) $\int_0^4 \sqrt{4t+1}\, dt$

(B) $2\int_0^4 \sqrt{t^2+1}\, dt$

(C) $\int_0^4 \sqrt{2t^2+1}\, dt$

(D) $\int_0^4 \sqrt{4t^2+1}\, dt$

(E) $2\pi\int_0^4 \sqrt{4t^2+1}\, dt$

---

24. Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g'(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$

(D) $\frac{f'(x)g(x) - f(x)g'(x)}{\left(f(x)\right)^2}$

(E) nonexistent

---

25. Consider the curve in the xy-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

(A) 20.086 (B) 0.342 (C) –0.005 (D) –0.011 (E) –0.033

26. If $y = \arctan(e^{2x})$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\dfrac{2e^{2x}}{1+e^{4x}}$ (C) $\dfrac{e^{2x}}{1+e^{4x}}$ (D) $\dfrac{1}{\sqrt{1-e^{4x}}}$ (E) $\dfrac{1}{1+e^{4x}}$

---

27. The interval of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \le 3$ (B) $-3 \le x \le 3$ (C) $-2 < x < 4$

(D) $-2 \le x < 4$ (E) $0 \le x \le 2$

---

28. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2 + 2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

(A) $\left(\dfrac{3}{4}, 8\right)$ (B) $\left(\dfrac{3}{4}, 4\right)$ (C) $\left(\dfrac{1}{8}, 8\right)$ (D) $\left(\dfrac{1}{8}, 4\right)$ (E) $\left(-\dfrac{5}{16}, 4\right)$

---

29. $\int x \sec^2 x \, dx =$

(A) $x \tan x + C$ (B) $\dfrac{x^2}{2} \tan x + C$ (C) $\sec^2 x + 2\sec^2 x \tan x + C$

(D) $x \tan x - \ln|\cos x| + C$ (E) $x \tan x + \ln|\cos x| + C$

---

30. What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \dfrac{\pi}{3}$?

(A) $\dfrac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\dfrac{8\pi}{3}$

(E) $\pi \ln\left(\dfrac{1}{2} + \sqrt{3}\right)$

31. If $s_n = \left(\dfrac{(5+n)^{100}}{5^{n+1}}\right)\left(\dfrac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\dfrac{1}{5}$ (B) 1 (C) $\dfrac{5}{4}$ (D) $\left(\dfrac{5}{4}\right)^{100}$ (E) The sequence does not converge.

---

32. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \left(f(x) + g(x)\right) dx = 4$

III. $\int_a^b \left(f(x)g(x)\right) dx = -5$

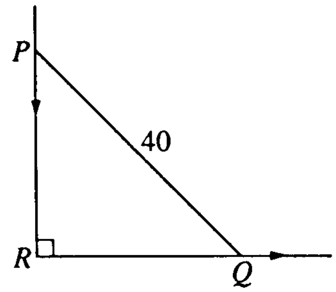
(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

---

33. Which of the following is equal to $\int_0^\pi \sin x\, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ (B) $\int_0^\pi \cos x\, dx$ (C) $\int_{-\pi}^0 \sin x\, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$ (E) $\int_\pi^{2\pi} \sin x\, dx$



34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

---

35. If F and f are differentiable functions such that $F(x) = \int_0^x f(t)dt$, and if $F(a) = -2$ and $F(b) = -2$ where $a < b$, which of the following must be true?

(A) $f(x) = 0$ for some x such that $a < x < b$.

(B) $f(x) > 0$ for all x such that $a < x < b$.

(C) $f(x) < 0$ for all x such that $a < x < b$.

(D) $F(x) \le 0$ for all x such that $a < x < b$.

(E) $F(x) = 0$ for some x such that $a < x < b$.

---

36. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

37. If $f(x)=\begin{cases} x & \text{for } x\le 1 \\ \dfrac{1}{x} & \text{for } x>1, \end{cases}$ then $\int_0^e f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e+\frac{1}{2}$

---

38. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

---

39. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

---

40. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y=\ln(1+2x-x^2)$. If Simpson's Rule with 2 subintervals is used to approximate the area of R, the approximation is

(A) 0.462 (B) 0.693 (C) 0.924 (D) 0.986 (E) 1.850

---

41. Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

---

42. $\lim_{x\to 0}(1+2x)^{\csc x} =$

(A) 0 (B) 1 (C) 2 (D) e (E) e^2

43. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin\left(x^2\right)$ is

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

---

44. If f is continuous on the interval $[a,b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

---

45. If $f(x) = \sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

1997 AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2
(B) 4
(C) 6
(D) 36
(E) 42

---

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

(A) $\dfrac{3x-3}{\sqrt{2x-3}}$

(B) $\dfrac{x}{\sqrt{2x-3}}$

(C) $\dfrac{1}{\sqrt{2x-3}}$

(D) $\dfrac{-x+3}{\sqrt{2x-3}}$

(E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

---

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

(A) $a + 2b + 5$ (B) $5b - 5a$ (C) $7b - 4a$ (D) $7b - 5a$ (E) $7b - 6a$

---

4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) –1 (D) –3 (E) –5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

---

6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

---

7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

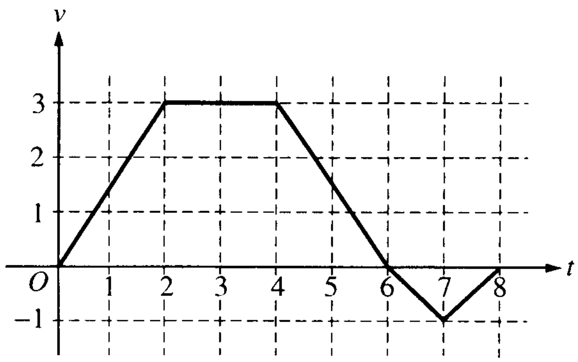
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

---

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

---

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \dfrac{\pi}{4}$ is

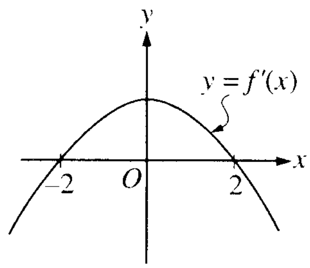
(A) $y - 1 = -\left(x - \dfrac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \dfrac{\pi}{4}\right)$

(C) $y = 2\left(x - \dfrac{\pi}{4}\right)$

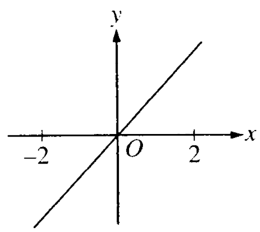
(D) $y = -\left(x - \dfrac{\pi}{4}\right)$

(E) $y = -2\left(x - \dfrac{\pi}{4}\right)$

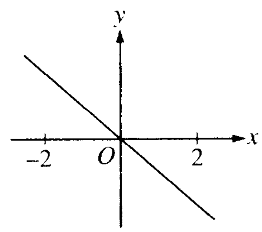


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

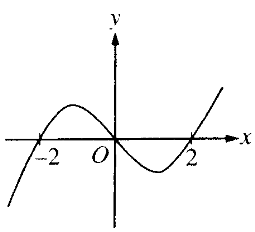
(A)



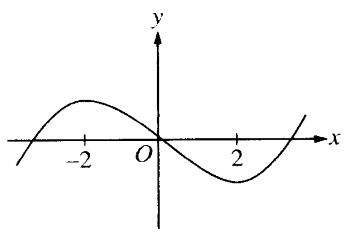
(B)



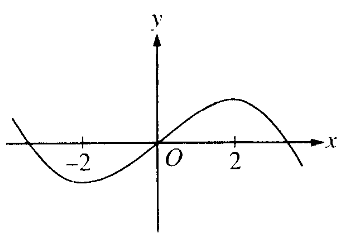
(C)



(D)



(E)



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12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

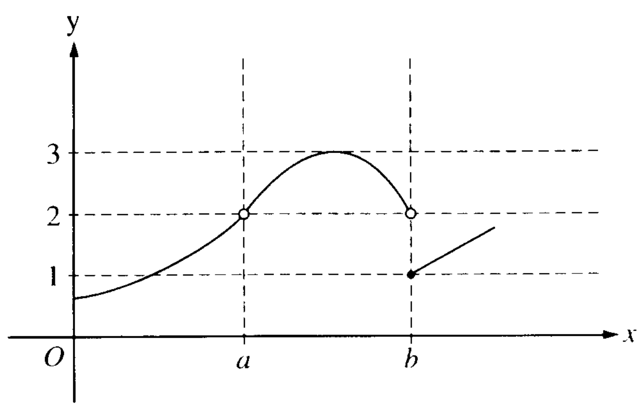
13. Let f be a function defined for all real numbers x. If $f'(x)=\dfrac{\left|4-x^2\right|}{x-2}$, then f is decreasing on the interval

(A) $(-\infty,2)$ (B) $(-\infty,\infty)$ (C) $(-2,4)$ (D) $(-2,\infty)$ (E) $(2,\infty)$

---

14. Let f be a differentiable function such that $f(3)=2$ and $f'(3)=5$. If the tangent line to the graph of f at $x=3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x)=\lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x)=2$

(C) $\lim_{x\to b} f(x)=2$

(D) $\lim_{x\to b} f(x)=1$

(E) $\lim_{x\to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

---

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

---

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is

(A) 0 (B) 1 (C) $e - 1$ (D) e (E) $e + 1$

---

19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2 - 1}\right|$

(B) $\frac{2x}{\left|x^2 - 1\right|}$

(C) $\frac{2|x|}{x^2 - 1}$

(D) $\frac{2x}{x^2 - 1}$

(E) $\frac{1}{x^2 - 1}$

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

---

21. $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

---

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

---

23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{5}$ (B) $\dfrac{16\pi}{3}$ (C) $\dfrac{16\pi}{5}$ (D) $\dfrac{8\pi}{3}$ (E) π

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\cdots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

---

25. $\int x\sin(2x)\,dx =$

(A) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+C$

(B) $-\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(C) $\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(D) $-2x\cos(2x)+\sin(2x)+C$

(E) $-2x\cos(2x)-4\sin(2x)+C$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

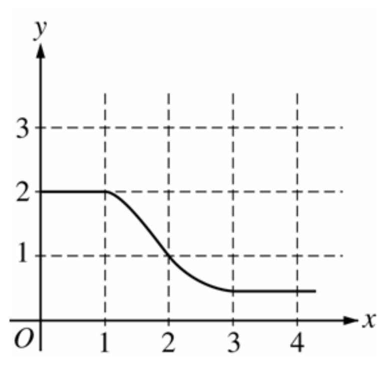
(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

---

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) –1.58 (B) –1.63 (C) –1.67 (D) –1.89 (E) –2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

---

80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $\left(x, f(x)\right)$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

---

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

---

82. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) –16 (B) –8 (C) –4 (D) 0 (E) 2

---

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

---

84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) –0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

---

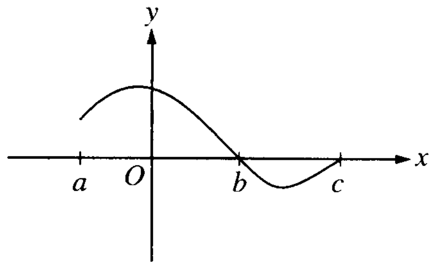
86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

---

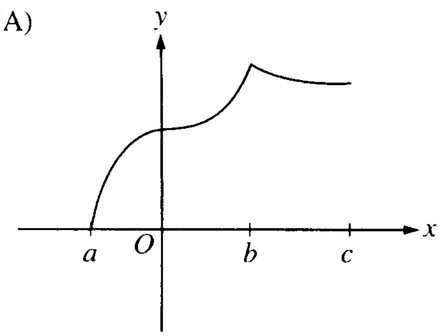
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

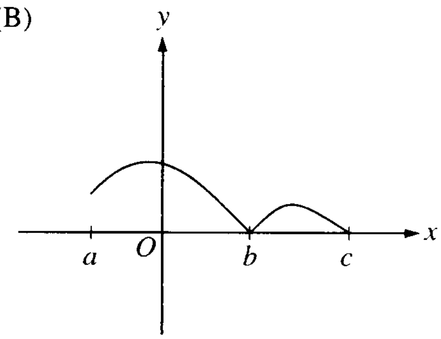


88. Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

(A)



(B)



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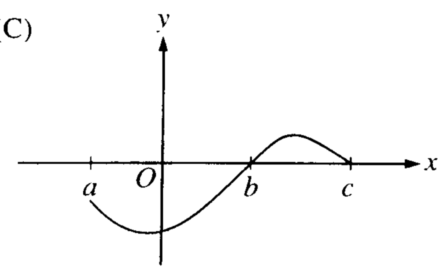

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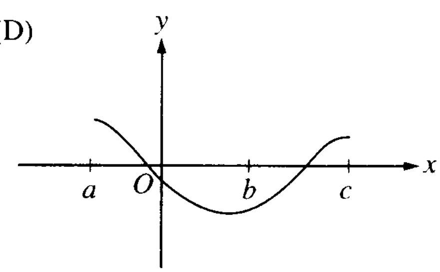
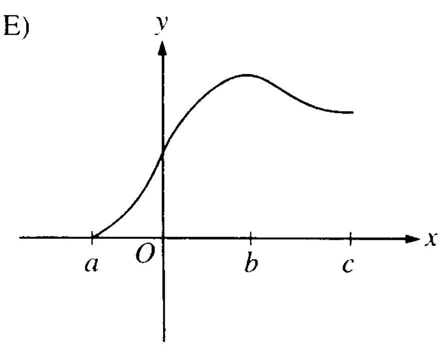


(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

---

90. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

1997 AP Calculus BC: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^1 \sqrt{x}\left(x+1\right) dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

---

2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

---

3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

---

4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

---

5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

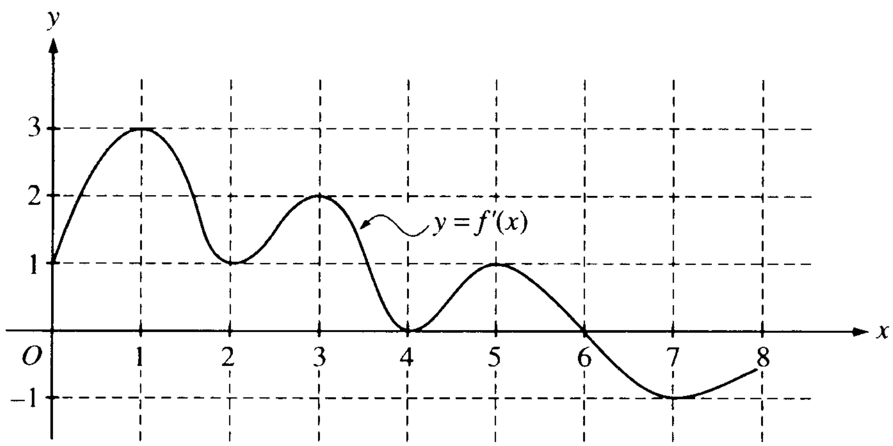
(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

---

6. The line normal to the curve $y = \sqrt{16-x}$ at the point $(0,4)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8

Questions 7-9 refer to the graph and the information below.



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

7. The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x - 3)$

(D) $y + 5 = 2(x - 3)$

(E) $y + 5 = 2(x + 3)$

---

8. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of x does the absolute minimum of f occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

---

10. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

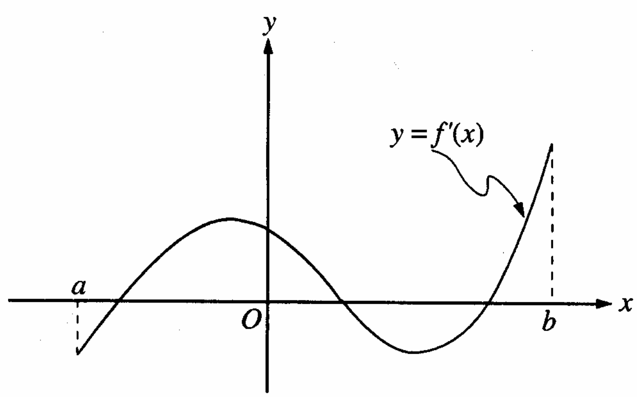
(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

---

11. $\int_1^\infty \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

---



12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

13. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

14. The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1{,}024} + \ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

---

15. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

---

16. $\lim_{h \to 0} \frac{e^h - 1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

17. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

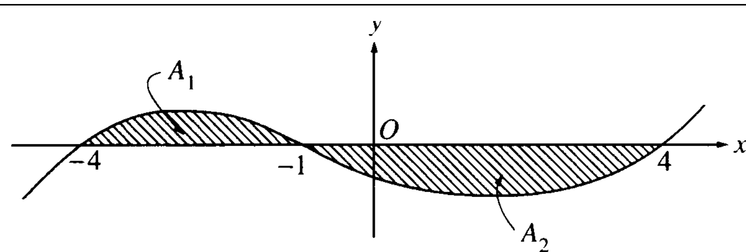
(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

---

18. For what values of t does the curve given by the parametric equations $x=t^3-t^2-1$ and $y=t^4+2t^2-8t$ have a vertical tangent?

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value



19. The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx-2\int_{-1}^{4} f(x)\,dx=$

(A) A_1 (B) A_1-A_2 (C) $2A_1-A_2$ (D) A_1+A_2 (E) A_1+2A_2

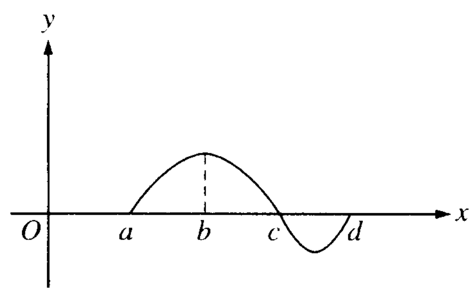
20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

(A) $-3 \le x \le 3$
(B) $-3 < x < 3$
(C) $-1 < x \le 5$
(D) $-1 \le x \le 5$
(E) $-1 \le x < 5$

---

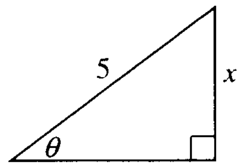
21. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta\, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$ (E) $3\int_0^{\pi} \cos\theta\, d\theta$



22. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\, dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.



23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

---

24. The Taylor series for $\sin x$ about $x = 0$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$. If f is a function such that $f'(x) = \sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

---

25. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\, \Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. Which of the following sequences converge?

I. $\left\{\dfrac{5n}{2n-1}\right\}$

II. $\left\{\dfrac{e^n}{n}\right\}$

III. $\left\{\dfrac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

77. When the region enclosed by the graphs of $y = x$ and $y = 4x - x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(B) $\pi\int_0^3 \left(x^2 - \left(4x - x^2\right)^2\right) dx$

(C) $\pi\int_0^3 \left(3x - x^2\right)^2 dx$

(D) $2\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(E) $2\pi\int_0^3 \left(3x^2 - x^3\right) dx$

78. $\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \frac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$

---

79. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

---

80. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

---

81. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one c between –3 and 6

(C) $-1 \le f(x) \le 3$ for all x between –3 and 6

(D) $f(c) = 1$ for at least one c between –3 and 6

(E) $f(c) = 0$ for at least one c between –1 and 3

82. If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\,dt \ge \int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

---

83. If $\frac{dy}{dx} = (1 + \ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$
(B) $1 + \ln x$
(C) $\ln x$
(D) $e^{2x + x\ln x - 2}$
(E) $e^{x \ln x}$

---

84. $\int x^2 \sin x\, dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$
(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$
(C) $-x^2 \cos x + 2x \sin x + 2\cos x + C$
(D) $-\frac{x^3}{3} \cos x + C$
(E) $2x \cos x + C$

---

85. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\frac{5}{2}$.

II. The average value of f is $\frac{9}{2}$.

III. The average value of f' is $\frac{5}{2}$.

(A) None
(B) I only
(C) III only
(D) I and III only
(E) II and III only

86. $\int \frac{dx}{(x-1)(x+3)} =$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

---

87. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

88. Let $f(x)=\int_0^{x^2}\sin t\,dt$. At how many points in the closed interval $\left[0,\sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

---

89. If f is the antiderivative of $\dfrac{x^2}{1+x^5}$ such that $f(1)=0$, then $f(4)=$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

---

90. A force of 10 pounds is required to stretch a spring 4 inches beyond its natural length. Assuming Hooke's law applies, how much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

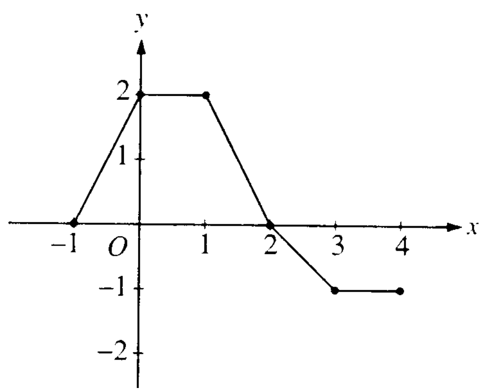
(A) 60.0 inch-pounds
(B) 45.0 inch-pounds
(C) 40.0 inch-pounds
(D) 15.0 inch-pounds
(E) 7.2 inch-pounds

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) –5 (E) -10



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

---

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)dx$ exists.

---

5. $\int_0^x \sin t \, dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

---

6. If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

---

7. $\int_1^e \left(\dfrac{x^2 - 1}{x}\right) dx =$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

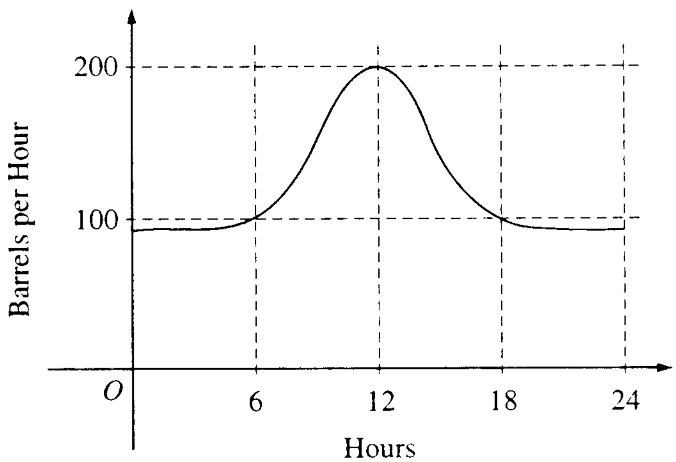
---

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

---

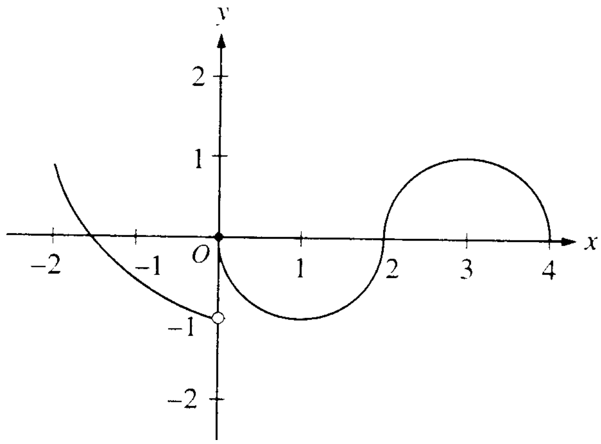
11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

---

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

---

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

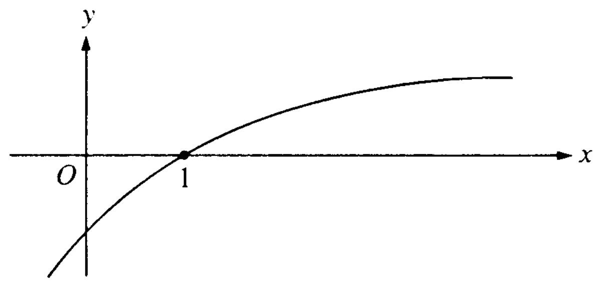
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

---

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

---

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) –1 only (B) 2 only (C) –1 and 0 only (D) –1 and 2 only (E) –1, 0, and 2 only

---

20. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) –3 (B) 0 (C) 3 (D) –3 and 3 (E) –3, 0, and 3

---

21. If $\dfrac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

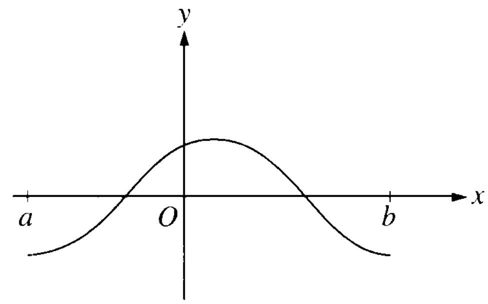
(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

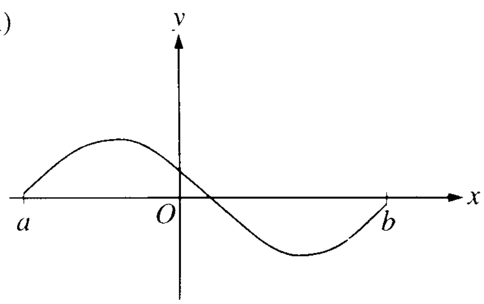
(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

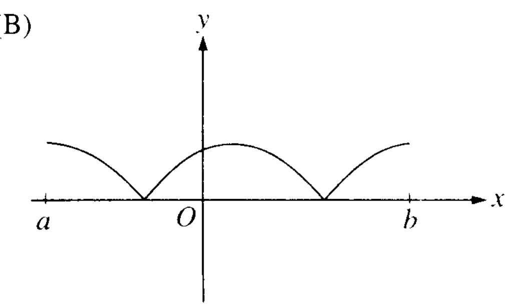


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

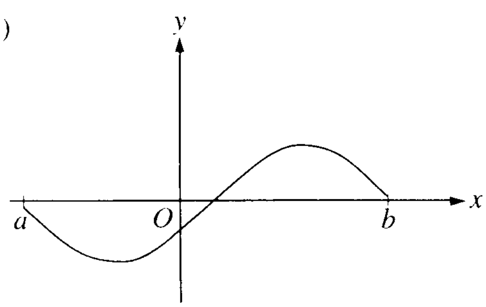
(A)



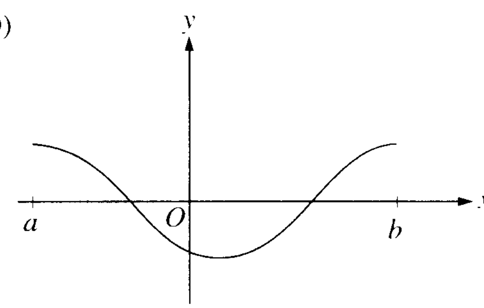
(B)



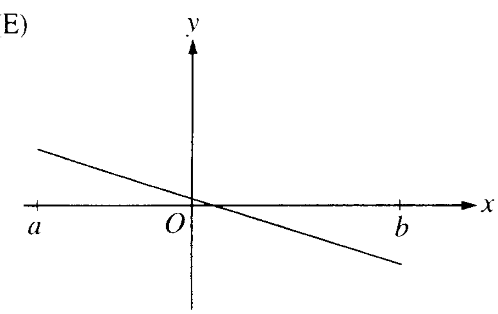
(C)



(D)



(E)



24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

---

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

---

27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

---

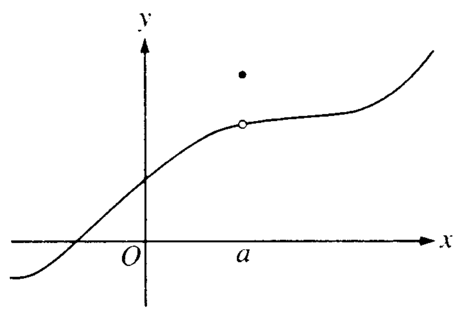
28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

---

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

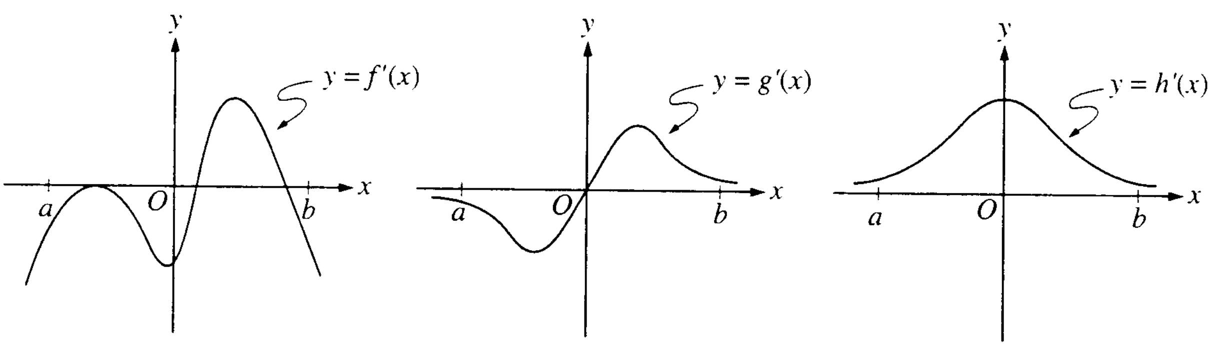
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

---

80. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true?

I. f is continuous at $x=0$.
II. f is differentiable at $x=0$.
III. f has an absolute minimum at $x=0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

82. If f is a continuous function and if $F'(x)=f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3)-2F(1)$

(B) $\frac{1}{2}F(3)-\frac{1}{2}F(1)$

(C) $2F(6)-2F(2)$

(D) $F(6)-F(2)$

(E) $\frac{1}{2}F(6)-\frac{1}{2}F(2)$

---

83. If $a \neq 0$, then $\lim\limits_{x\to a} \dfrac{x^2-a^2}{x^4-a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

---

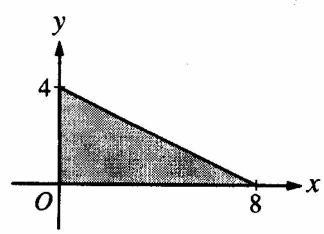
84. Population y grows according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

---

87. Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

---

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

---

91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5,\ f(5) = 5,$ and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some $c,\ 2 < c < 5,\ f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

---

92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

---

2. In the *xy*-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

---

3. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

---

4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

(E) $\ln\left|(x-2)(x-4)\right| + C$

5. If f and g are twice differentiable and if $h(x) = f\left(g(x)\right)$, then $h''(x) =$

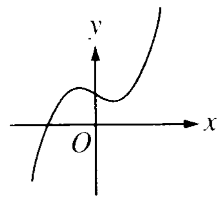
(A) $f''\left(g(x)\right)\left[g'(x)\right]^2 + f'\left(g(x)\right)g''(x)$

(B) $f''\left(g(x)\right)g'(x) + f'\left(g(x)\right)g''(x)$

(C) $f''\left(g(x)\right)\left[g'(x)\right]^2$

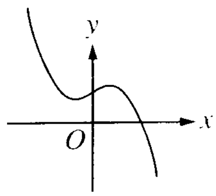
(D) $f''\left(g(x)\right)g''(x)$

(E) $f''\left(g(x)\right)$

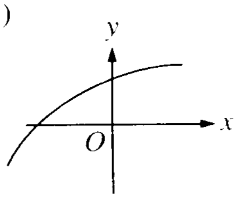


6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

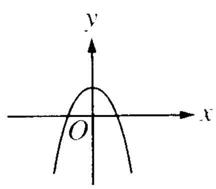
(A)



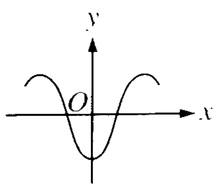
(B)



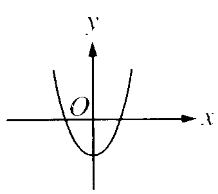
(C)



(D)



(E)



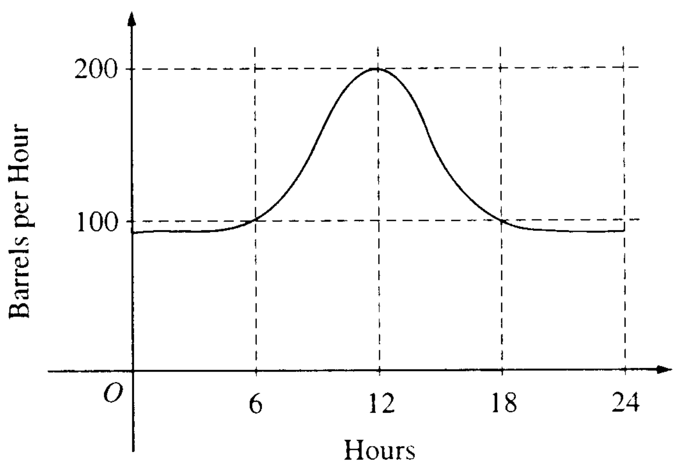
7. $\int_{1}^{e}\left(\frac{x^2-1}{x}\right)dx =$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

---

8. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t = 1$ is

(A) $(0,1)$ (B) $(2,3)$ (C) $(2,6)$ (D) $(6,12)$ (E) $(6,24)$

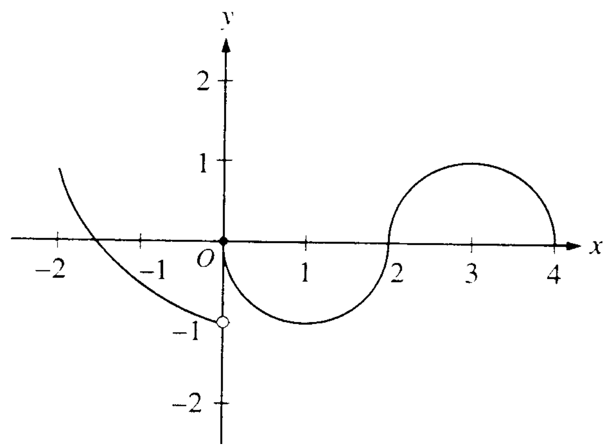
---

11. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

12. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2\ln 2 & \text{for } 2<x\le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

15. $\int x \cos x \, dx =$

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

(C) $-x \sin x + \cos x + C$

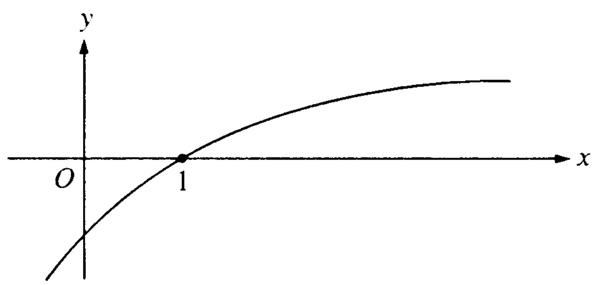
(D) $x \sin x + C$

(E) $\frac{1}{2} x^2 \sin x + C$

---

16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

---

19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2 \, d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4\sin\theta - 2)^2 \, d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin\theta - 2)^2 \, d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(16\sin^2\theta - 4\right) d\theta$ (E) $\frac{1}{2}\int_0^{\pi} \left(16\sin^2\theta - 4\right) d\theta$

---

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

---

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1} \, dt$

(B) $\int_0^1 \sqrt{t^2 + t} \, dt$

(C) $\int_0^1 \sqrt{t^4 + t^2} \, dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4} \, dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9} \, dt$

22. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges

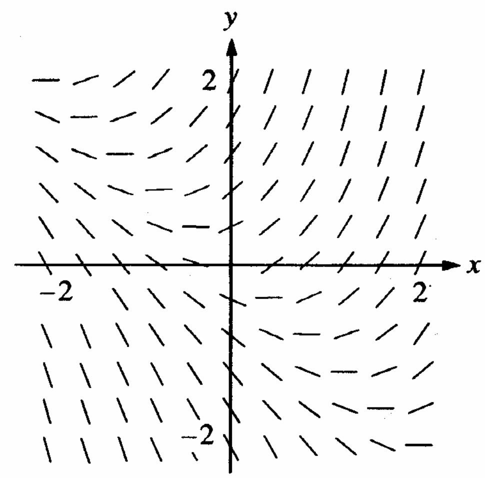
(E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ diverges

---

23. Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative maximum at c and $a<c<b$, which of the following statements must be true?

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c)=0$.
III. If $f''(c)$ exists, then $f''(c)\le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

---

25. $\int_0^{\infty} x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

---

26. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

---

27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n{}^{n-1}$

---

28. $\lim_{x \to 1} \frac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

1998 AP Calculus BC: Section I, Part B

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

---

77. If f is a vector-valued function defined by $f(t) = \left(e^{-t}, \cos t\right)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$

(B) $e^{-t} - \cos t$

(C) $\left(-e^{-t}, -\sin t\right)$

(D) $\left(e^{-t}, \cos t\right)$

(E) $\left(e^{-t}, -\cos t\right)$

---

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

79. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

---

80. Let R be the region enclosed by the graph of $y=1+\ln\left(\cos^4 x\right)$, the x-axis, and the lines $x=-\dfrac{2}{3}$ and $x=\dfrac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

81. If $\dfrac{dy}{dx}=\sqrt{1-y^2}$, then $\dfrac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\dfrac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\dfrac{1}{2}$

---

82. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 \left[f(x)+g(x)\right]dx=$

(A) $2\int_3^5 g(x)\,dx+7$

(B) $2\int_3^5 g(x)\,dx+14$

(C) $2\int_3^5 g(x)\,dx+28$

(D) $\int_3^5 g(x)\,dx+7$

(E) $\int_3^5 g(x)\,dx+14$

83. The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

---

84. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n}}$ converges?

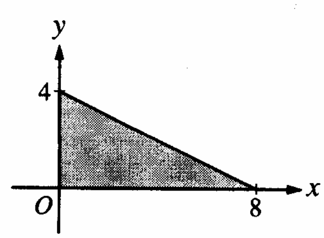
(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

---

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

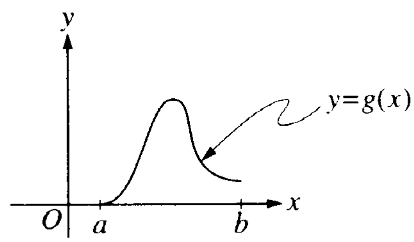


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 2y = 8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

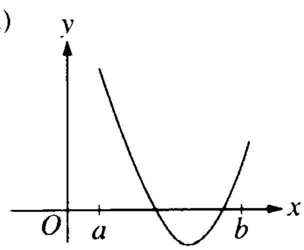
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

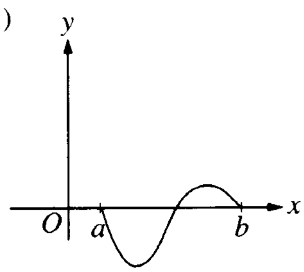
87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

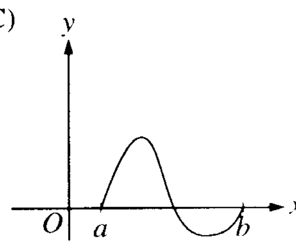
(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

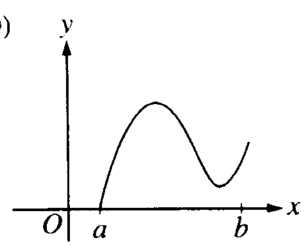


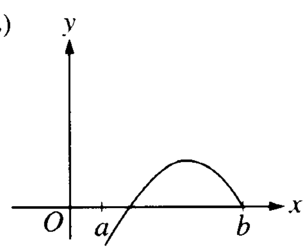
88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


(B)


(C)


(D)


(E)


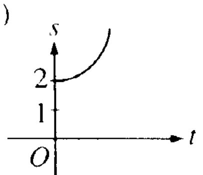
89. The graph of the function represented by the Maclaurin series

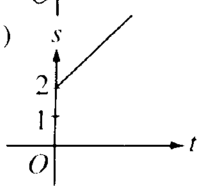
$1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + \ldots + \dfrac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

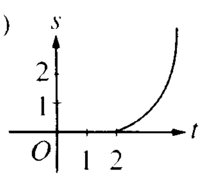
(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

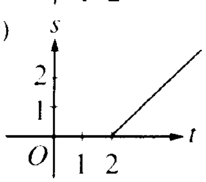
---

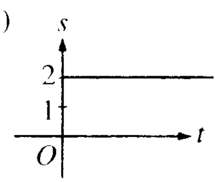
90. A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

(A)


(B)


(C)


(D)


E)


t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

---

92. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8