enjoy getting the right answers so much that they only review the stuff they know. The time to concentrate on what you know is when you are taking the test.

- Practice writing free-response answers. The College Board publishes copies of student answers from past years. If your teacher has some of these, look at them and learn what is expected and what is not needed.

- Plan your review carefully. Don't try to cram the weekend before the exam. The day before the test: relax, get psyched, and get a good night's sleep. On the day of the test eat a good breakfast. The test is grueling, even though you're up for it. Bring a snack for the brief break between the multiple-choice and free-response sections.

Calculators

The reason calculators are so important in learning mathematics is that they allow you do the graphical and numerical work easily, quickly and accurately. You should use your calculator all year, on homework, tests and when studying. Learn how to use it efficiently. Learn its strengths and weaknesses.

You may use your calculator any way you wish. There are four types of things you should definitely know how to do. They are:

- Plot the graph of a function within an arbitrary viewing window,
- Find the zeros of functions (solve equations numerically),
- Calculate the numerical value of a derivative at a point, and
- Calculate the numerical value of a definite integral.

You may have programs in your calculator, but you will not be asked to use them. The questions on the exam are designed so that someone with a program, or a more expensive calculator, has no advantage over someone who does not. This includes many of the built-in programs.

Be sure your calculator is set in Radian mode.

Numerical answers may be left unsimplified and in terms of π, e, etc. There is no reason to change an answer to a decimal if you don't have to. (Why take the chance of pushing the wrong button?)

Install fresh batteries before the exam.

The Format of the Exams

There are two parts to the AP Exams: a multiple-choice section and a free-response section. The number of questions and timing may change slightly from year to year. Be sure you check the current College Board publications for your exam.

Both sections count equally towards your final grade. Both sections cover the full range of topics. It is natural to expect that different classes will cover some topics in greater detail than others; the exam will evaluate your knowledge of the calculus. It is not necessary to answer all the questions to get a good score. In fact, the exam is made so that the average score will be

about 50%, this is usually a score of 3. You are probably used to class averages much higher than 50%; this test is different. Expect not to be able to answer some questions, and don't worry about it. Use your time on the ones you can answer.

The Current AP Calculus Exam Format is

Section I Part A (55 minutes) 28 multiple-choice questions for which you may not use a calculator.

Section I Part B (50 minutes) 17 multiple-choice questions. You may use your calculator on this section. Some of these questions require the use of a graphing calculator, others do not.

Section II Part A (45 minutes) Three free-response questions. You may use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may continue work on this section without a calculator after you start part B.

Section II Part B (45 minutes) Three free-response questions. You may not use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may use part of this time to work on Section II, Part A without a calculator

Multiple Choice Questions

Read each question carefully and look at the answer choices. Do the ones you are sure of. Don't struggle over one that isn't working out. Remember your time is limited and you do not need to answer all of the questions. There is a penalty for guessing, so don't guess blindly. You receive one point for each correct answer. One-quarter point is deducted for each wrong answer. Nothing is deducted for a question that is left blank. Guessing may improve your score only if you can eliminate one or more of the choices. Be sure to bubble your answer in the correct space on the answer sheet.

Types of Multiple-choice Questions

- One type of question may ask for a computation (a limit, a derivative, a definite or indefinite integral) and give five possible answers. Be aware that answers which result from predictable mistakes are among the choices — work carefully, just because your answer is there doesn't mean it's correct.

- Another type may ask you only to set up a problem. Looking at the answer choices may keep you from doing too much work.

- Some questions ask you to choose the one true or one false statement from a list of five statements. Be sure you know if you are looking for a true or a false statement.

- Another type of question asks which of three statements is true (or false). The answer may be any one or some combination of the statements.

- Another type may ask you to choose the correct table or graph from among five choices.

Free-response Questions

The general directions for Section II require you to show your work and indicate the methods

- Ignoring units of measure.

- Family of function problems: Questions that start with a phrase like, "This question deals with functions defined by $f(x) = 1 + b\sin(x)$ where b is a positive constant..." are meant to be done in general, not for a specific value of b. Even if you get the correct answer using a specific value of b, you may lose points. The reason is that, because you used a particular value, you have no way to be sure that your answers are true for all values of b.

- Don't Curve Fit: Occasionally, a function is given as a graph or a table of values with no equation. You are being asked to demonstrate that you can work from the graphical or numerical data. The questions that follow can be answered without an equation. You may have learned to approximate functions using various curve fitting (regression) operations built into your calculator. This should be avoided. While this is a perfectly good approach in the real world, you may lose points because you are not working with the function you were given, (only an approximation of it), and curve fitting is not one of the four allowed calculator operations.

- Using a built-in calculator utility or a program without showing all the work and justification for what you are doing. You may do only the four things listed on page ii without further explanation.

A Word About Three-Decimal Place Accuracy.

Some answers, the evaluation of definite integrals is a prime example, must be written as decimals because they are found using a graphing calculator. These answers, and other answers that you choose to change to decimals, must be correct to three places past the decimal point. This means that the answer may be rounded to three decimal places, truncated after the third decimal place, or left with more than three decimal places, as long as the first three are correct. An answer of π, which should be left as π, may be given as 3.1415926535898..., 3.142, 3.141, or even 3.14199999. If the number ends in zeros, they may be omitted; thus, 17.320 may be given as 17.32, and 56.000 may be given as 56.

Too often students may choose to give decimal answers when they are not required. Once a free-response answer is entirely in terms of numbers, there is no need to change the number to a decimal. For example, 1999 AB 1(c) does not require a decimal answer: $-\frac{1}{2}\cos 4 + \frac{7}{2}$ is sufficient. If the decimal is correct (to three decimal places) you will receive the credit. However, if you change a correct answer to an incorrect decimal (including one with too few decimals) then you will lose credit. The moral is: avoid arithmetic, avoid decimals; give them only if you cannot give anything else.

Rounding too soon is another common mistake made by students. Computations should be done with more decimal places than is required in the final answer. Learn how to store the intermediate values in your calculator, and recall them when you need them in a computation. If premature rounding affects the three decimal place accuracy of the final answer, you will not be given the answer point. However, a rounded answer used in the next part of a problem will not be held against you.

Good Luck!

Sample Examination I

<u>Section I Part A</u>

<u>Directions</u>: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may NOT be used on this part of the exam.

<u>In this test</u>: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If f is a continuous function defined by $f(x) = \begin{cases} x^2 + bx, & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right), & x > 5 \end{cases}$ then $b =$

(A) -6

(B) -5

(C) -4

(D) 4

(E) 5

Answer

2. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) $(0,0)$ only

(B) $(1,2)$ only

(C) $(2,4)$ only

(D) $(4,-16)$ only

(E) $(0,0)$ and $(2,4)$

Answer

3. A line through the origin, rotates around the origin in such a way that the angle, θ, between the line and the positive x-axis changes at the rate of $\frac{d\theta}{dt}$ for time $t \geq 0$. Which expression gives the rate at which the slope of the line is changing?

(A) $\frac{d\theta}{dt}$ (B) $\cos\theta\frac{d\theta}{dt}$ (C) $-\sin\theta\frac{d\theta}{dt}$

(D) $\frac{1}{\cos^2\theta}\frac{d\theta}{dt}$ (E) $\tan\theta\frac{d\theta}{dt}$

Answer

4. If $f(x) = e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0, 2\pi]$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

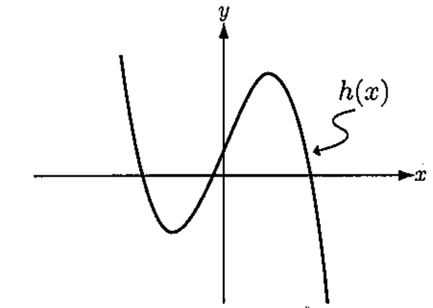
Answer

5. $\lim\limits_{x\to\infty} \dfrac{10^8x^5 + 10^6x^4 + 10^4x^2}{10^9x^6 + 10^7x^5 + 10^5x^3} =$

(A) 0 (B) 1 (C) -1 (D) $\frac{1}{10}$ (E) $-\frac{1}{10}$

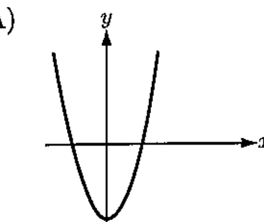
Answer

6.

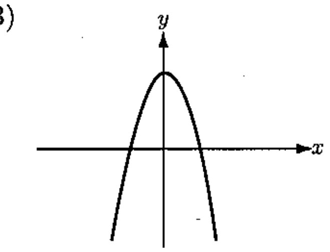


The graph of $h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

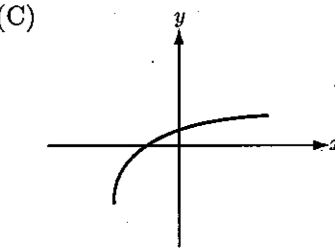
(A)



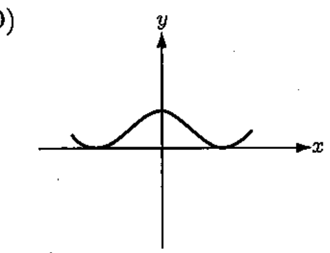
(B)



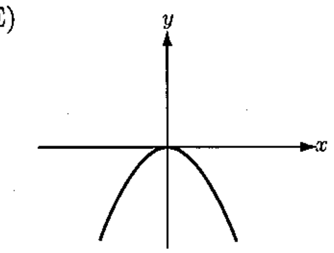
(C)



(D)



(E)



Answer

7. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) =$

(A) -2

(B) 0

(C) 1

(D) $\frac{\sqrt{2}}{2}$

(E) $\sqrt{2}$

Answer

8. If t is measured in hours and $f'(t)$ is measured in knots, then $\int_0^2 f'(t)\,dt =$
(Note: 1 knot = 1 nautical mile/hour)

(A) $f(2)$ knots

(B) $f(2) - f(0)$ knots

(C) $f(2)$ nautical miles

(D) $f(2) - f(0)$ nautical miles

(E) $f(2) - f(0)$ knots/ hour

Answer

9. The equation of the tangent line to the curve $x^2 + y^2 = 169$ at the point $(5, -12)$ is

(A) $5y - 12x = -120$

(B) $5x - 12y = 119$

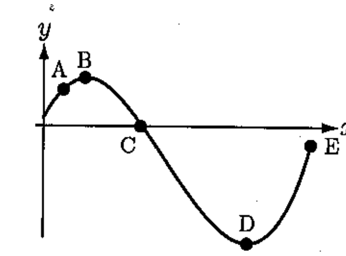
(C) $5x - 12y = 169$

(D) $12x + 5y = 0$

(E) $12x + 5y = 169$

Answer

10.



The figure above shows the graph of the velocity of a moving object as a function of time. At which of the marked points is the speed the greatest?

(A) A

(B) B

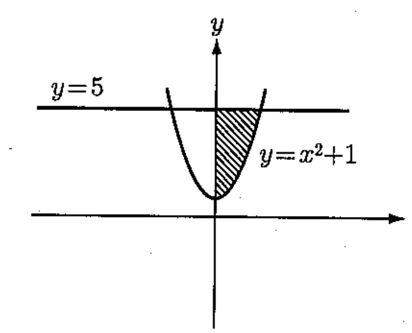
(C) C

(D) D

(E) E

Answer

11.



For the figure above, the area of the shaded region is

(A) $\frac{14}{3}$

(B) $\frac{16}{3}$

(C) $\frac{28}{3}$

(D) $\frac{32}{3}$

(E) $\frac{65}{3}$

Answer

12. $\int \frac{1}{\sqrt{4-x^2}}\,dx =$

(A) $\text{Arcsin}\,\frac{x}{2}+C$ (B) $2\sqrt{4-x^2}+C$ (C) $\text{Arcsin}\,x+C$

(D) $\sqrt{4-x^2}+C$ (E) $\frac{1}{2}\text{Arcsin}\frac{x}{2}+C$

Answer

13. If the graph of $f(x)=2x^2+\frac{k}{x}$ has a point of inflection at $x=-1$, then the value of k is

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

Answer

14. $\int \sin(3x+4)\,dx =$

(A) $-\frac{1}{3}\cos(3x+4)+C$

(B) $-\cos(3x+4)+C$

(C) $-3\cos(3x+4)+C$

(D) $\cos(3x+4)+C$

(E) $\frac{1}{3}\cos(3x+4)+C$

Answer

15. What are all values of x for which the graph of $y=\frac{2}{4-x}$ is concave downward?

(A) No values of x

(B) $x<4$

(C) $x>-4$

(D) $x<-4$

(E) $x>4$

Answer

16. A particle moves along the x-axis in such a way that its position at time t is given by $x(t)=\frac{1-t}{1+t}$. What is the acceleration of the particle at time $t=0$?

(A) -4

(B) -2

(C) $-\frac{3}{5}$

(D) 2

(E) 4

Answer

17. If, for all real number x, $f(x) = g(x) + 5$, then on any interval $[a, b]$ the area of the region between the graphs of f and g is

(A) 5

(B) $5a + 5b$

(C) $5b - 5a$

(D) $5a - 5b$

(E) $5ab$

Answer

18. If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$

(A) $3(\ln x)^2$

(B) $(\ln x)(2x + \ln x)$

(C) $(\ln x)(2 + \ln x)$

(D) $(\ln x)(2 + x\ln x)$

(E) $(\ln x)(1 + \ln x)$

Answer

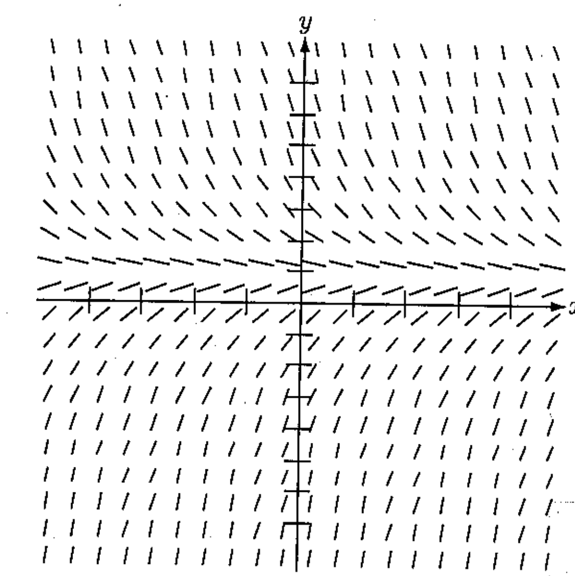
19. If $\int_0^6 (x^2 - 2x + 2)\,dx$ is approximated by three inscribed rectangles of equal width on the x-axis, then the approximation is

(A) 24 (B) 26 (C) 28

(D) 48 (E) 76

Answer

20.



Shown above is the slope field for which differential equation?

(A) $\frac{dy}{dx} = 1 - x$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = -\frac{x}{y}$

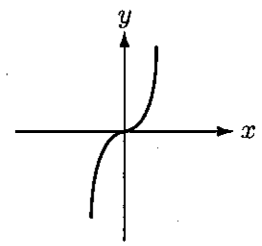
(D) $\frac{dy}{dx} = \frac{y}{x}$

(E) $\frac{dy}{dx} = 1 - y$

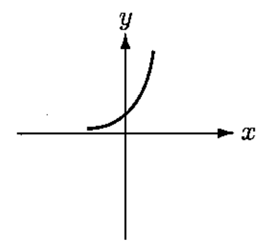
Answer

21. If, for all real numbers x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph f?

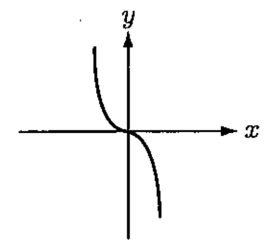
(A)



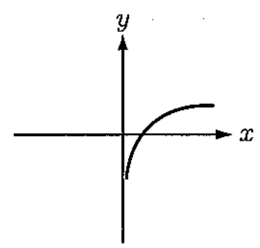
(B)



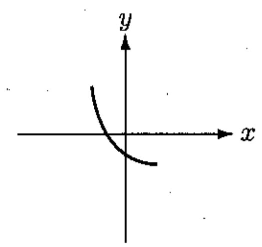
(C)



(D)



(E)



Answer

22. If $\frac{dy}{dx} = x^2y^2$, then $\frac{d^2y}{dx^2} =$

(A) $2xy^2$

(B) $4x^3y^3$

(C) $2x + 2x^2y^3$

(D) $2x^2y + 2xy^2$

(E) $2x^4y^3 + 2xy^2$

Answer

23. $4\int_1^{e^2} \frac{x - x^3}{x^2}\, dx =$

(A) $3 - e^2$

(B) $3 - e^4$

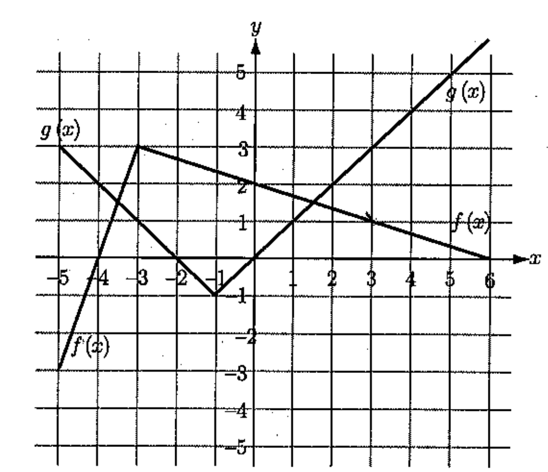
(C) $5 - e^2$

(D) $5 - e^4$

(E) $10 - 2e^4$

Answer

24.



The functions f and g are piecewise linear functions whose graph are shown above. If $h(x) = f(x)g(x)$, then $h'(3) =$

(A) $-\frac{8}{3}$

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{2}{3}$

(E) $\frac{8}{3}$

Answer

25. For which pair of functions $f(x)$ and $g(x)$ below, will the $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$?

	$f(x)$	$g(x)$
(A)	e^x	x^2
(B)	e^x	$\ln x$
(C)	$\ln x$	e^x
(D)	x	$\ln x$
(E)	3^x	2^x

Answer

26. Let $f(x)$ be the function defined by $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$.

The value of $\int_{-2}^{1} x f(x)\, dx =$

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) 3

(D) $\frac{7}{2}$

(E) $\frac{11}{2}$

Answer

27. The average value of the function $f(x) = \cos\left(\frac{1}{2}x\right)$ on the closed interval $[-4, 0]$ is

(A) $-\frac{1}{2}\sin(2)$

(B) $-\frac{1}{4}\sin(2)$

(C) $\frac{1}{2}\cos(2)$

(D) $\frac{1}{4}\sin(2)$

(E) $\frac{1}{2}\sin(2)$

Answer

28. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+(1/n)} + \frac{1}{1+(2/n)} + \cdots + \frac{1}{1+(n/n)}\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x}\, dx$

(B) $\int_1^2 \frac{1}{x+1}\, dx$

(C) $\int_1^2 x\, dx$

(D) $\int_1^2 \frac{2}{x+1}\, dx$

(E) $\int_1^2 \frac{1}{x}\, dx$

Answer

Section I Part B

Directions: Solve each of the following problems, using available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem. Calculators may be used on this part of the exam.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices, the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The volume of the solid formed by revolving the region bounded by the graph of $y = (x-3)^2$ and the coordinate axes about the x–axis is given by which of the following integrals?

(A) $\pi \int_0^3 (x-3)^2 \, dx$

(B) $\pi \int_0^3 (x-3)^4 \, dx$

(C) $2\pi \int_0^3 (x-3)^2 \, dx$

(D) $2\pi \int_0^3 x(x-3)^2 \, dx$

(E) $2\pi \int_0^3 x(x-3)^4 \, dx$

Answer

30. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^2$. At what value of x in the interval $0 \leq x \leq \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0

(B) 0.660

(C) 2.083

(D) 2.194

(E) 2.207

Answer

31. Let $f(t) = \frac{1}{t}$ for $t > 0$. For what value of t is $f'(t)$ equal to the average rate of change of f on $[a, b]$?

(A) $-\sqrt{ab}$

(B) $\sqrt{ab}$

(C) $-\frac{1}{\sqrt{ab}}$

(D) $\frac{1}{\sqrt{ab}}$

(E) $\sqrt{\frac{1}{2}\left(\frac{1}{b} - \frac{1}{a}\right)}$

Answer

32.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	20	19.5	18	15.5	12	7.5	2	−4.5	−12	−20.5	−30

Some values of a continuous function are given in the table above. Using 10 subintervals of equal length, the Trapezoidal Rule approximation for $\int_0^{10} f(x)\,dx$ is

(A) 7.500

(B) 32.500

(C) 33.325

(D) 33.333

(E) 57.500

Answer

33. Let $R(t)$ represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water in gallons that leaks out in the first three hours?

(A) $R(3) - R(0)$

(B) $\dfrac{R(3) - R(0)}{3 - 0}$

(C) $\int_0^3 R(t)\,dt$

(D) $\int_0^3 R'(t)\,dt$

(E) $\frac{1}{3}\int_0^3 R(t)\,dt$

Answer

34. Let f and g be differentiable functions such that

$$f(1) = 4, g(1) = 3, f'(3) = -5$$
$$f'(1) = -4, g'(1) = -3, g'(3) = 2$$

If $h(x) = f(g(x))$, then $h'(1) =$

(A) −9

(B) 15

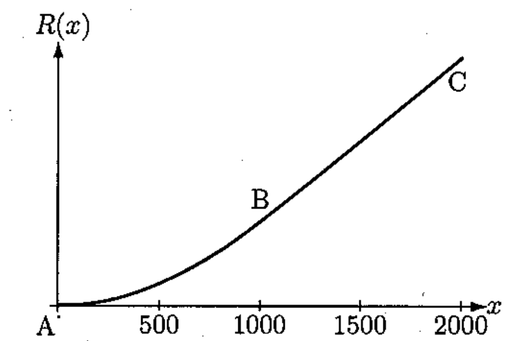
(C) 0

(D) −5

(E) −12

Answer

35.



The figure above shows a road running in the shape of a parabola from the bottom of a hill at A to point B. At B it changes to a line and continues on to C. The equation of the road is

$$R(x) = \begin{cases} ax^2, & \text{from A to B} \\ bx + c, & \text{from B to C} \end{cases}$$

B is 1000 feet horizontally from A and 100 feet higher. Since the road is smooth, $R'(x)$ is continuous. What is the value of b?

(A) 0.2

(B) 0.02

(C) 0.002

(D) 0.0002

(E) 0.00002

Answer

36. The area of the region enclosed by the graphs of $y = e^{(x^2)} - 2$ and $y = \sqrt{4 - x^2}$ is

(A) 2.525

(B) 4.049

(C) 4.328

(D) 5.050

(E) 6.289

Answer

37.

t(sec)	0	2	4	6	8
$a(t)$ (ft/sec^2)	2	3	4	3	2

The table for the acceleration of a particle from 0 to 8 seconds is given in the table above. If the velocity at $t = 0$ is 4 feet per second, the approximation value of the velocity, in feet per second, at $t = 8$ seconds, computed using the Riemann Sum with four subdivisions of equal length is

(A) 4

(B) 12

(C) 16

(D) 24

(E) 28

Answer

38. Suppose that $f(x)$ is an <u>even</u> function and let $\int_0^1 f(x)\,dx = 5$ and $\int_0^7 f(x)\,dx = 1$.

What is $\int_{-7}^{-1} f(x)\,dx$?

(A) -5

(B) -4

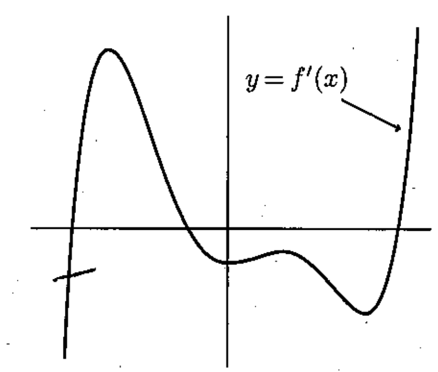
(C) 0

(D) 4

(E) 5

Answer

39.



The figure above shows the graph of the derivative of a function f. How many points of inflection does f have in the interval shown?

(A) None

(B) One

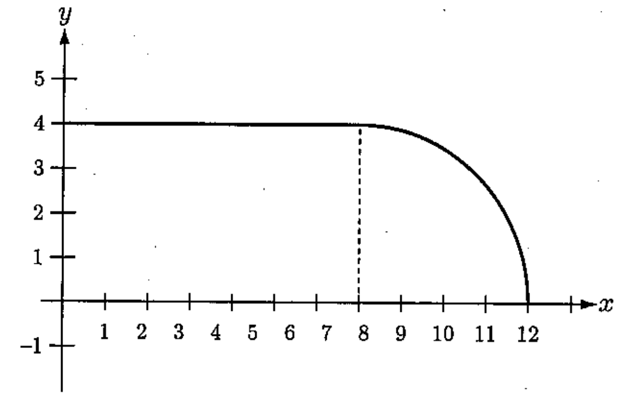
(C) Two

(D) Three

(E) Four

Answer

40.



As shown in the figure above the function $f(x)$ consists of a line segment from $(0,4)$ to $(8,4)$ and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval $[0,12]$?

(A) 2

(B) 3.714

(C) 3.9

(D) 22.283

(E) 41.144

Answer

41. If f is the function defined by $f(x) = \sqrt[3]{x^2+4x}$ and g is an antiderivative of f such that $g(5) = 7$, then $g(1) \approx$

(A) −3.882

(B) −3.557

(C) 1.710

(D) 3.557

(E) 3.882

Answer

42. The amount $A(t)$ of a certain item produced in a factory is given by

$$A(t) = 4000 + 48(t-3) - 4(t-3)^3$$

where t is the number of hours of production since the beginning of the workday at 8:00 am. At what time is the rate of production increasing most rapidly?

(A) 8:00 am (B) 10:00 am (C) 11:00 am

(D) 12:00 noon (E) 1:00 pm

Answer

43. At how many points on the curve $y = 4x^5 - 3x^4 + 15x^2 + 6$ will the line tangent to the curve pass through the origin?

(A) One (B) Two (C) Three

(D) Four (E) Five

Answer

44. A population grows according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$, t measured in years. This population will approach a limiting value as time goes on. During which year will the population reach half of this limiting value?

(A) Second

(B) Third

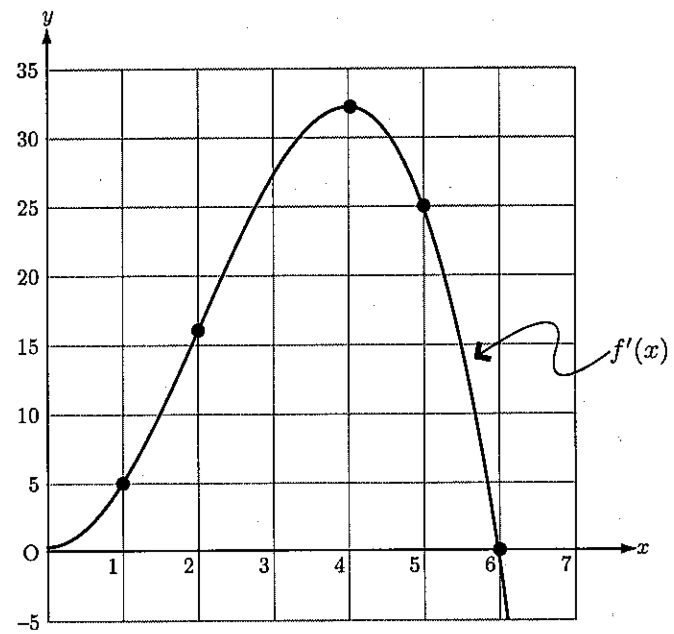
(C) Fourth

(D) Eighth

(E) Twenty-ninth

Answer

45.



Note: This is the graph of $f'(x)$, NOT the graph of $f(x)$.

Let f be a differentiable function for all x. The graph of $f'(x)$ is shown above. If $f(2) = 10$, which of the following best approximates the maximum value of $f(x)$?

(A) 30

(B) 50

(C) 70

(D) 90

(E) 110

Answer

SECTION II - FREE-RESPONSE QUESTIONS
GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART IN THE PINK TEST BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numerical or algebraic) need not be simplified.
- If you use decimal approximations in calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: QUESTIONS 1,2,3

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS IN THIS SECTION OF THE EXAM.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

YOU HAVE 45 MINUTES FOR PART A.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

SECTION II PART B: QUESTIONS 4,5,6

YOU MAY **NOT** USE A GRAPHING CALCULATOR IN THIS PART OF THE EXAM.

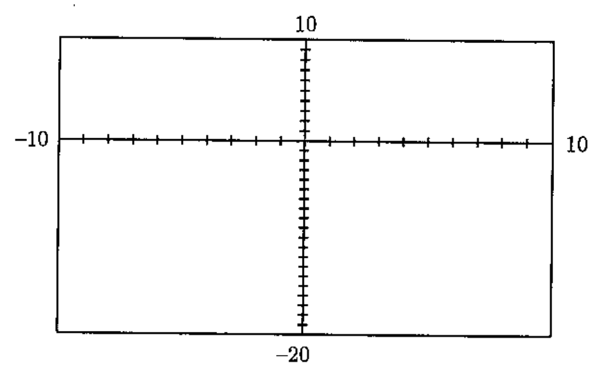
YOU HAVE 45 MINUTES FOR PART B.

YOU MAY USE SOME OF THIS TIME TO WORK ON PART A. IF YOU WORK ON PART A, YOU MAY **NOT** USE A GRAPHING CALCULATOR.

Section II Part A: Graphing Calculator MAY BE USED.

1. Let f be the function given by $f(x) = \dfrac{x^2 - 8}{x + 3}$ for all $x \neq -3$ and whose derivative is given by $f'(x) = \dfrac{x^2 + 6x + 8}{(x+3)^2}$ for all $x \neq -3$.

   (a) In the viewing window provided below, sketch the graph of f.



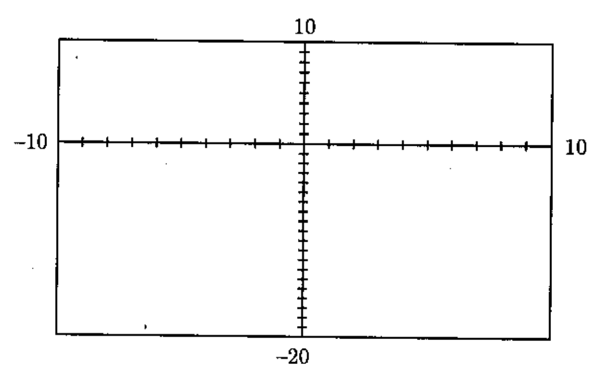
Viewing Window
$[-10, 10]$ by $[-20, 10]$

   (b) Find the range of f. Use $f'(x)$ to justify your answer.

   (c) Find $\lim_{x \to \pm\infty} f'(x)$.

   (d) Explain what your answer to (c) tells about the graph of $f(x)$.

(a) In the viewing window provided below, sketch the graph of f.



Viewing Window
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(d) Explain what your answer to (c) tells about the graph of $f(x)$.

Section II Part A: Graphing Calculator MAY BE USED.

2. Let $v(t) = 3t^2 - 12t$ be the velocity of a particle moving along the x-axis for time t, $t \geq 0$. When $t = 0$, the particle is at $x = -6$.

(a) Determine the position of the particle at $t = a$.

(b) Write an expression for the speed of the particle at any time t.

(c) In which direction does the particle begin moving and when does it turn around?

(d) When, if ever, is the particle at the origin?

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(d) When, if ever, is the particle at the origin?

Section II Part A: Graphing Calculator MAY BE USED.

3. The number of minutes of daylight per day, $L(d)$, at 40° North latitude is modeled by the function

$$L(d) = 167.5 \sin\left(\frac{2\pi}{366}(d-80)\right) + 731$$

where d is the number of days after the beginning of 1996. (For Jan 1, 1996, $d = 1$; and for Dec. 31, 1996, $d = 366$ since 1996 was a leap year.)

(a) Which day (d) has the most minutes of daylight? Justify your answer.

(b) What is the average (mean) number of minutes of daylight in 1996? Justify your answer.

(c) What is the total number of minutes of daylight in 1996? Justify your answer.

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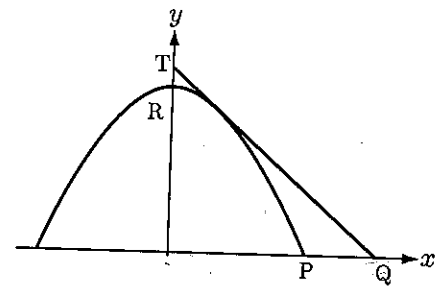
Section II Part B: Graphing Calculator MAY **NOT** BE USED.

4.



Note: Figure not drawn to scale.

Suppose that the graph of $y = 100 - \frac{x^2}{400}$ represents a hill rising from level ground represented by the x-axis as shown in the figure above. Let x and y be measured in feet. A 16-foot tower, TR, stands on the top of the hill.

(a) Determine the equation of $\overleftrightarrow{TQ}$, the line of sight from the top of the tower tangent to the hill. Justify your answer.

(b) From the top of the tower, the ground between P and Q is blocked by the hill and cannot be seen. Find the distance between P and Q to the nearest foot.

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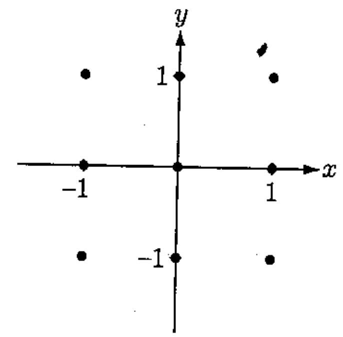
Section II Part B: Graphing Calculator MAY **NOT** BE USED.

5. Consider the differential equation $\frac{dy}{dx} = xy^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) Find the general solution of the given differential equation in terms of a constant C.

(c) Find the particular solution of the differential condition that satisfies the initial condition $y(0) = 1$.

(d) For what values of the constant C will the solutions of the differential equation have one or more vertical asymptotes? Justify your answer.

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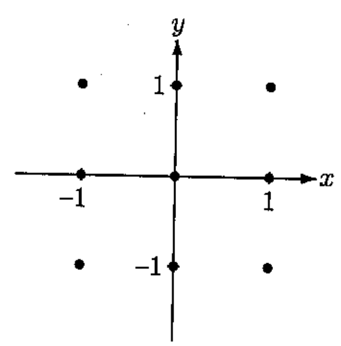

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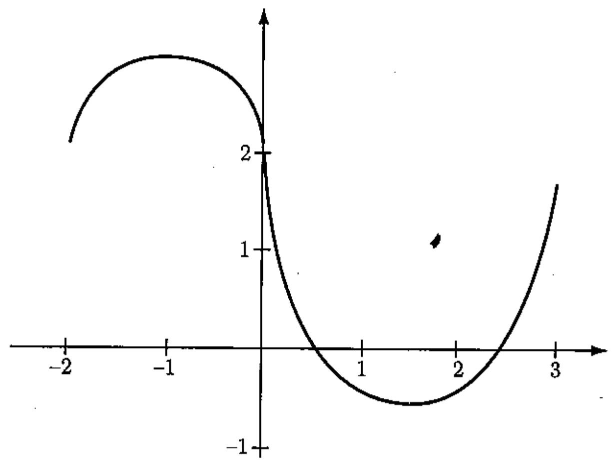
Section II Part B: Graphing Calculator MAY **NOT** BE USED.

6.



The graph of $f(x)$

Let f be a continuous function defined on the closed interval $[-2, 3]$. The graph of f consists of a semicircle and a semi-ellipse, as shown above. Let $G(x) = G(-2) + \int_{-2}^{x} f(t)\, dt$.

(a) On what intervals, if any, is G concave down? Justify your answer.

(b) If the equation of the line tangent to the graph of $G(x)$ at the point where $x = 0$ is $y = mx + 7$, what is the value of m and the value of $G(0)$? Justify your answer.

(c) If the average value of f on the interval $0 \le x \le 3$ is zero, find the value of $G(3)$. Show your work that leads to your answer.

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