

Sample AP Calculus AB and BC Exam Questions

The sample exam questions that follow illustrate the relationship between the course framework and the AP Calculus AB and BC Exams and serve as examples of the types of questions that appear on the exams. After the sample questions is a table that shows which skill, learning objective(s), and unit each question relates to. The table also provides the answers to the multiple-choice questions.

Section I: Multiple-Choice

PART A (AB OR BC)

Graphing calculators are not permitted on this part of the exam.

1. $\lim_{x \rightarrow 0} \frac{1 - \cos^2(2x)}{(2x)^2} =$
- (A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1

$$f(x) = \begin{cases} \frac{2}{x} & \text{for } x < -1 \\ x^2 - 3 & \text{for } -1 \leq x \leq 2 \\ 4x - 3 & \text{for } x > 2 \end{cases}$$

2. Let f be the function defined above. At what values of x , if any, is f not differentiable?
- (A) $x = -1$ only
(B) $x = 2$ only
(C) $x = -1$ and $x = -2$
(D) f is differentiable for all values of x .

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-4	-5	3
2	-3	1	8	4

3. The table above gives values of the differentiable functions f and g and their derivatives at selected values of x . If h is the function defined by $h(x) = f(x)g(x) + 2g(x)$, then $h'(1) =$
- (A) 32
 (B) 30
 (C) -6
 (D) -16
4. If $x^3 - 2xy + 3y^2 = 7$, then $\frac{dy}{dx} =$
- (A) $\frac{3x^2 + 4y}{2x}$
 (B) $\frac{3x^2 - 2y}{2x - 6y}$
 (C) $\frac{3x^2}{2x - 6y}$
 (D) $\frac{3x^2}{2 - 6y}$
5. The radius of a right circular cylinder is increasing at a rate of 2 units per second. The height of the cylinder is decreasing at a rate of 5 units per second. Which of the following expressions gives the rate at which the volume of the cylinder is changing with respect to time in terms of the radius r and height h of the cylinder?
 (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)
- (A) $-20\pi r$
 (B) $-2\pi r h$
 (C) $4\pi r h - 5\pi r^2$
 (D) $4\pi r h + 5\pi r^2$

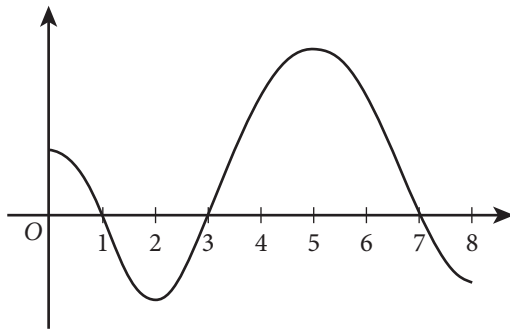
6. Which of the following is equivalent to the definite integral $\int_2^6 \sqrt{x} \, dx$?

(A) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4}{n} \sqrt{\frac{4k}{n}}$

(B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{6}{n} \sqrt{\frac{6k}{n}}$

(C) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4}{n} \sqrt{2 + \frac{4k}{n}}$

(D) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{6}{n} \sqrt{2 + \frac{6k}{n}}$



Graph of g

7. The figure above shows the graph of the continuous function g on the interval $[0, 8]$. Let h be the function defined by $h(x) = \int_3^x g(t) \, dt$. On what intervals is h increasing?

(A) $[2, 5]$ only

(B) $[1, 7]$

(C) $[0, 1]$ and $[3, 7]$

(D) $[1, 3]$ and $[7, 8]$

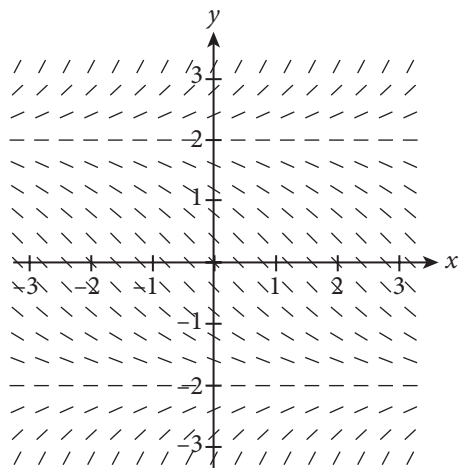
8. $\int \frac{x}{\sqrt{1-9x^2}} \, dx =$

(A) $-\frac{1}{9} \sqrt{1-9x^2} + C$

(B) $-\frac{1}{18} \ln \sqrt{1-9x^2} + C$

(C) $\frac{1}{3} \arcsin(3x) + C$

(D) $\frac{x}{3} \arcsin(3x) + C$



9. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y-2}{2}$

(B) $\frac{dy}{dx} = \frac{y^2-4}{4}$

(C) $\frac{dy}{dx} = \frac{x-2}{2}$

(D) $\frac{dy}{dx} = \frac{x^2-4}{4}$

10. Let R be the region bounded by the graph of $x = e^y$, the vertical line $x = 10$, and the horizontal lines $y = 1$ and $y = 2$. Which of the following gives the area of R ?

(A) $\int_1^2 e^y dy$

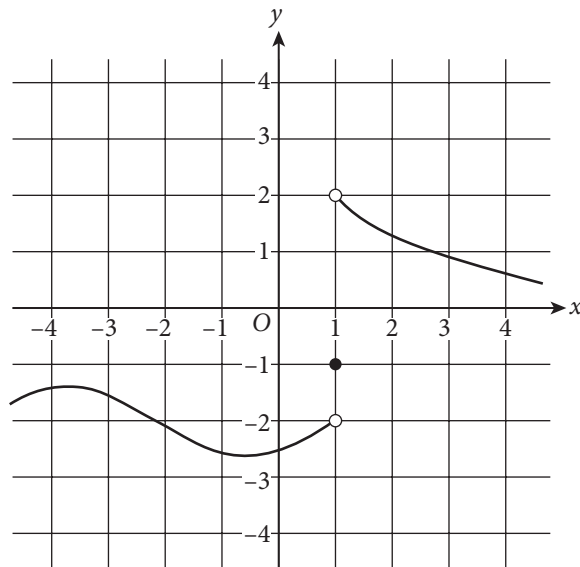
(B) $\int_e^{e^2} \ln x dx$

(C) $\int_1^2 (10 - e^y) dy$

(D) $\int_e^{10} (\ln x - 1) dx$

PART B (AB OR BC)

A graphing calculator is required on this part of the exam.



Graph of f

11. The graph of the function f is shown in the figure above. The value of $\lim_{x \rightarrow 1^+} f(x)$ is
- (A) -2
 - (B) -1
 - (C) 2
 - (D) nonexistent
12. The velocity of a particle moving along a straight line is given by $v(t) = 1.3t \ln(0.2t + 0.4)$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 1.2$?
- (A) -0.580
 - (B) -0.548
 - (C) -0.093
 - (D) 0.660

x	-1	0	2	4	5
$f'(x)$	11	9	8	5	2

13. Let f be a twice-differentiable function. Values of f' , the derivative of f , at selected values of x are given in the table above. Which of the following statements must be true?
- (A) f is increasing for $-1 \leq x \leq 5$.
- (B) The graph of f is concave down for $-1 < x < 5$.
- (C) There exists c , where $-1 < c < 5$, such that $f'(c) = -\frac{3}{2}$.
- (D) There exists c , where $-1 < c < 5$, such that $f''(c) = -\frac{3}{2}$.
14. Let f be the function with derivative defined by $f'(x) = 2 + (2x - 8)\sin(x + 3)$. How many points of inflection does the graph of f have on the interval $0 < x < 9$?
- (A) One
- (B) Two
- (C) Three
- (D) Four
15. Honey is poured through a funnel at a rate of $r(t) = 4e^{-0.35t}$ ounces per minute, where t is measured in minutes. How many ounces of honey are poured through the funnel from time $t = 0$ to time $t = 3$?
- (A) 0.910
- (B) 1.400
- (C) 2.600
- (D) 7.429

PART A (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

x	2	5
$f(x)$	4	7
$f'(x)$	2	3

16. The table above gives values of the differentiable function f and its derivative f' at selected values of x .
- If $\int_2^5 f(x) dx = 14$, what is the value of $\int_2^5 x \cdot f'(x) dx$?
- (A) 13
- (B) 27
- (C) $\frac{63}{2}$
- (D) 41

17. The number of fish in a lake is modeled by the function F that satisfies the logistic differential equation $\frac{dF}{dt} = 0.04F \left(1 - \frac{F}{5000} \right)$, where t is the time in months and $F(0) = 2000$. What is $\lim_{t \rightarrow \infty} F(t)$?

- (A) 10,000
- (B) 5000
- (C) 2500
- (D) 2000

18. A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = \sin(t^2)$.

Which of the following is an expression for $\frac{d^2y}{dx^2}$ in terms of t ?

- (A) $-\sin(t^2)$
- (B) $-2t \sin(t^2)$
- (C) $\cos(t^2) - 2t^2 \sin(t^2)$
- (D) $2\cos(t^2) - 4t^2 \sin(t^2)$

19. Which of the following series is conditionally convergent?

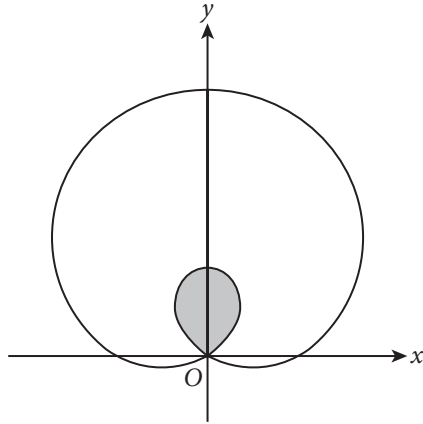
- (A) $\sum_{k=1}^{\infty} (-1)^k \frac{5}{k^3 + 1}$
- (B) $\sum_{k=1}^{\infty} (-1)^k \frac{5}{k + 1}$
- (C) $\sum_{k=1}^{\infty} (-1)^k \frac{5k}{k + 1}$
- (D) $\sum_{k=1}^{\infty} (-1)^k \frac{5k^2}{k + 1}$

20. Let f be the function defined by $f(x) = e^{2x}$. Which of the following is the Maclaurin series for f' , the derivative of f ?

- (A) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$
- (B) $2 + 2x + \frac{2x^2}{2!} + \frac{2x^3}{3!} + \cdots + \frac{2x^n}{n!} + \cdots$
- (C) $1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots + \frac{(2x)^n}{n!} + \cdots$
- (D) $2 + 2(2x) + \frac{2(2x)^2}{2!} + \frac{2(2x)^3}{3!} + \cdots + \frac{2(2x)^n}{n!} + \cdots$

PART B (BC ONLY)

A graphing calculator is required on this part of the exam.



21. The figure above shows the graph of the polar curve $r = 2 + 4\sin \theta$. What is the area of the shaded region?
- (A) 2.174
(B) 2.739
(C) 13.660
(D) 37.699
22. The function f has derivatives of all orders for all real numbers. It is known that $|f^{(4)}(x)| \leq \frac{12}{5}$ and $|f^{(5)}(x)| \leq \frac{3}{2}$ for $0 \leq x \leq 2$. Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 2$. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $|f(2) - P_4(2)| \leq k$?
- (A) $\frac{2^5}{5!} \cdot \frac{3}{2}$
(B) $\frac{2^5}{5!} \cdot \frac{12}{5}$
(C) $\frac{2^4}{4!} \cdot \frac{3}{2}$
(D) $\frac{2^4}{4!} \cdot \frac{12}{5}$

Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

PART A (AB OR BC)

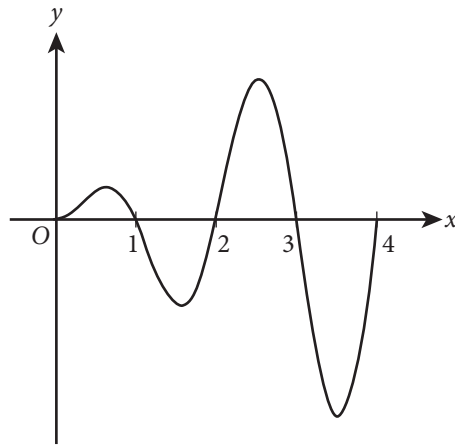
A graphing calculator is required on this part of the exam.

t (hours)	0	2	4	6	8	10	12
$R(t)$ (vehicles per hour)	2935	3653	3442	3010	3604	1986	2201

- On a certain weekday, the rate at which vehicles cross a bridge is modeled by the differentiable function R for $0 \leq t \leq 12$, where $R(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). Values of $R(t)$ for selected values of t are given in the table above.
 - Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.
 - Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t) dt$. Indicate units of measure.
 - On a certain weekend day, the rate at which vehicles cross the bridge is modeled by the function H defined by $H(t) = -t^3 - 3t^2 + 288t + 1300$ for $0 \leq t \leq 17$, where $H(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). According to this model, what is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \leq t \leq 12$?
 - For $12 < t < 17$, $L(t)$, the local linear approximation to the function H given in part (c) at $t = 12$, is a better model for the rate at which vehicles cross the bridge on the weekend day. Use $L(t)$ to find the time t , for $12 < t < 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

PART B (AB OR BC)

Graphing calculators are not permitted on this part of the exam.



Graph of f'

2. The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the closed interval $[0, 4]$. The areas of the regions bounded by the graph of f' and the x -axis on the intervals $[0, 1]$, $[1, 2]$, $[2, 3]$, and $[3, 4]$ are 2, 6, 10, and 14, respectively. The graph of f' has horizontal tangents at $x = 0.6$, $x = 1.6$, $x = 2.5$, and $x = 3.5$. It is known that $f(2) = 5$.
- On what open intervals contained in $(0, 4)$ is the graph of f both decreasing and concave down? Give a reason for your answer.
 - Find the absolute minimum value of f on the interval $[0, 4]$. Justify your answer.
 - Evaluate $\int_0^4 f(x)f'(x)dx$.
 - The function g is defined by $g(x) = x^3 f(x)$. Find $g'(2)$. Show the work that leads to your answer.

PART A (BC ONLY)

A graphing calculator is required on this part of the exam.

3. For $0 \leq t \leq 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $\frac{dx}{dt} = \sin\left(\frac{t}{t+3}\right)$ and $\frac{dy}{dt} = e^{\cos t}$.
- Write an equation for the line tangent to the curve at the point $(2, -7)$.
 - Find the y -coordinate of the position of the particle at time $t = 4$.
 - Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.
 - Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

PART B (BC ONLY)

Graphing calculators are not permitted on this part of the exam.

4. The Maclaurin series for the function f is given by

$$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \dots \text{ on its interval of convergence.}$$

- (a) Use the ratio test to determine the interval of convergence of the Maclaurin series for f . Show the work that leads to your answer.
- (b) The Maclaurin series for f evaluated at $x = \frac{1}{4}$ is an alternating series whose

terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using

the first two nonzero terms of this series is $\frac{15}{64}$. Show that this

approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

- (c) Let h be the function defined by $h(x) = \int_0^x f(t) dt$. Write the first three nonzero terms and the general term of the Maclaurin series for h .

Answer Key and Question Alignment to Course Framework

Multiple-Choice Question	Answer	Skill	Learning Objective	Unit
1	D	1.E	LIM-1.E	1
2	B	3.D	FUN-2.A	2
3	A	1.E	FUN-3.B	2
4	B	1.E	FUN-3.D	3
5	C	1.E	CHA-3.D	4
6	C	2.C	LIM-5.C	6
7	C	2.D	FUN-5.A	6
8	A	1.E	FUN-6.D	6
9	B	2.C	FUN-7.C	7
10	C	1.D	CHA-5.A	8
11	C	2.B	LIM-1.C	1
12	C	1.E	CHA-3.B	4
13	D	3.D	FUN-1.B	5
14	D	2.D	FUN-4.A	5
15	D	3.D	CHA-4.D	8
16	A	1.E	FUN-6.E	6
17	B	3.F	FUN-7.H	7
18	A	1.E	CHA-3.G	9
19	B	3.D	LIM-7.A	10
20	D	3.D	LIM-8.G	10
21	A	3.D	CHA-5.D	9
22	A	1.F	LIM-8.C	10

Free-Response Question	Skills	Learning Objective	Unit
1	1.D, 1.E, 2.B, 3.F	CHA-2.D, CHA-3.A, CHA-3.C, CHA-3.F, CHA-4.B, LIM-5.A	2, 4, 6, 8
2	1.C, 1.E, 2.B, 2.E, 3.B, 3.E	FUN-3.B, FUN-4.A, FUN-5.A, FUN-6.D	2, 5, 6
3	1.C, 1.D, 1.E, 2.B	CHA-3.G, FUN-8.B	9
4	1.D, 1.E, 3.B, 3.D, 3.E	LIM-7.A, LIM-7.B, LIM-8.D, LIM-8.G	10